

# Scattering from a Buried PEMC Cylinder Illuminated by a Normally Incident Plane Wave Propagating in Free Space

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**Abstract** — A rigorous solution is presented to the problem of scattering by a perfect electromagnetic conducting (PEMC) circular cylinder buried inside a dielectric half-space that is excited by a normally incident transverse magnetic (TM) plane wave propagating in free space. The plane wave incident on the planar interface separating the two media creates fields transmitting into the dielectric half-space becoming the known primary incident fields for the buried cylinder. When the fields scattered by the cylinder, in response to those fields incident on it, are incident at the interface, they generate fields reflected into the dielectric half-space and fields transmitted into free space. These fields, and the fields scattered by the cylinder are expressed in terms of appropriate cylindrical waves consisting of unknown expansion coefficients which are to be determined. Imposing boundary conditions at the surface of the cylinder and at a point on the planar interface, enables the evaluation of the unknown coefficients. This procedure is then replicated, by considering multiple reflections and transmissions at the planar interface, and multiple scattering by the cylinder, till a preset accuracy is obtained for the reflection coefficient at the particular point on the interface. The reflection coefficient at this point is then computed for cylinders of different sizes, to show how it varies with the PEMC admittance of the cylinder, its burial depth, and the permittivity of the dielectric half-space.

**Index Terms** — Buried PEMC circular cylinder, cylindrical wave expansions, multiple reflections, planar interface, rigorous solution.

## I. INTRODUCTION

Solutions to electromagnetic scattering by buried bodies may be used to detect pipes, tunnels, mines, and other like objects. Analytical and numerical solutions for locating two and three dimensional bodies are already available in the literature. For instance, solutions using various cylindrical wave expressions have been obtained in the literature by some authors, for cylinders buried in homogenous and/or nonhomogeneous ground, lossy earth, and quite a few rough surfaces [1-10]. Alternatively, numerical solutions for scattering from arbitrary shaped buried bodies have been obtained using the Method of Moments (MoM) [11-13].

The objective of this paper is to obtain a rigorous solution to scattering by a PEMC circular cylinder buried in a dielectric half space, when excited by a plane wave propagating in free space.

The fields scattered by the PEMC cylinder due to those primary fields incident on it are expressed in terms of cylindrical waves comprising of unknown expansion coefficients. When the fields scattered by the buried cylinder are incident at the interface, they generate fields which are reflected into the dielectric medium and transmitted into free space. These fields are also expressed in terms of suitable cylindrical waves involving unknown expansion coefficients. Enforcing the boundary conditions at the surface of the buried PEMC cylinder and at a specified point on the planar interface leads to the evaluation of the unknown expansion coefficients. This procedure is then replicated, by considering multiple reflections and multiple transmissions at the planar interface, and multiple scattering by the cylinder, till a preset accuracy is obtained for the reflection coefficient at the specified point on the

interface. It is worth noting that the method of finding the unknown expansion coefficients does not require any matrix inversion, making it quite computationally efficient. Details of the paper are given in the following sections.

## II. FORMULATION

An infinitely long PEMC circular cylinder of radius  $a$  is buried in a dielectric half-space having a relative permittivity of  $\epsilon_r$  while positioned at a distance  $d$  from the planar interface separating the dielectric half-space and the free space regions, as shown in Fig. 1. Consider a uniform plane electromagnetic wave of amplitude  $E_0$  propagating in free space, being incident on this planar interface.

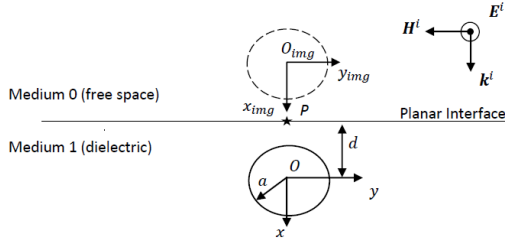


Fig. 1. Geometry of the buried PEMC cylinder located in a dielectric half-space.

For a TM polarized plane wave that is propagating in free space, the axial components of the incident electric and magnetic fields can be written as

$$E_z^i = E_0 e^{-jk_0 x} \quad (1)$$

$$H_y^i = -\frac{e^{-jk_0 x}}{\eta_0} \quad (2)$$

where  $k_0$  is the wavenumber and  $\eta_0$  is the wave impedance in free space.

The electric and magnetic field components of the reflected and transmitted waves generated due to the plane wave incident on the planar interface (in the absence of the cylinder), are given by

$$E_z^r = R E_0 e^{jk_0 x} \quad (3)$$

$$H_y^r = \frac{R E_0}{\eta_0} e^{jk_0 x} \quad (4)$$

$$E_z^t = T E_0 e^{-jk_1 x} \quad (5)$$

$$H_y^t = -\frac{T E_0}{\eta_1} e^{-jk_1 x} \quad (6)$$

where  $R$  and  $T$  are the reflected and transmitted field coefficients due to the presence of the planar interface and  $\eta_1$  is the intrinsic wave impedance in the dielectric half-space. These coefficients can be evaluated by enforcing the boundary conditions on the planar interface at  $x=-d$ , as

$$R = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} e^{2jk_0 d} \quad (7)$$

$$T = \frac{2\eta_1}{\eta_1 + \eta_0} e^{j(k_0 - k_1)d} \quad (8)$$

where  $k_1$  is the wavenumber in the dielectric half-space. The initial transmitted electric and magnetic fields corresponding to their components given by (5) and (6) will be incident on the PEMC cylinder. These field components may be written in terms of circular cylindrical wave functions in the dielectric medium as

$$E_z^t = \sum_{n=-\infty}^{\infty} A_n J_n(k_1 \rho) e^{jn\phi} \quad (9)$$

$$H_\phi^t = \frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} A_n J_n'(k_1 \rho) e^{jn\phi} \quad (10)$$

where  $J_n(k_1 \rho)$  is the first order Bessel function of order  $n$  and argument  $k_1 \rho$ , and the prime denotes its derivative with respect to the argument. The incident field expansion coefficients are given by  $A_n = (TE_0) j^{-n}$ . The scattered co-polar and cross polar field components due to the initial incident fields on the PEMC cylinder can be written as

$$E_z^{s,0} = \sum_{n=-\infty}^{\infty} B_n^0 H_n^{(2)}(k_1 \rho) e^{jn\phi} \quad (11)$$

$$H_\phi^{s,0} = \frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} B_n^0 H_n^{(2)'}(k_1 \rho) e^{jn\phi} \quad (12)$$

$$H_z^{s,0} = \frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} C_n^0 H_n^{(2)}(k_1 \rho) e^{jn\phi} \quad (13)$$

$$E_\phi^{s,0} = \sum_{n=-\infty}^{\infty} C_n^0 H_n^{(2)'}(k_1 \rho) e^{jn\phi} \quad (14)$$

where  $H_n^{(2)}(k_1 \rho)$  and  $H_n^{(2)'}(k_1 \rho)$  are the second kind of Hankel function and its derivative with respect to the argument, of order  $n$  and argument  $k_1 \rho$ , and  $B_n^0, C_n^0$  are the co-polar and cross polar scattered field coefficients, which can be found by applying the PEMC boundary condition on the

surface of the PEMC cylinder. The expressions for these coefficients are given by [14]

$$B_n^0 = -\frac{H_n^{(2)}(k_1 a) J_n'(k_1 a) + M^2 \eta_1^2 H_n^{(2)'}(k_1 a) J_n(k_1 a)}{(1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} A_n \quad (15)$$

$$C_n^0 = -\frac{2M \eta_1}{\pi k_1 a (1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} A_n \quad (16)$$

where  $M$  is the admittance of the PEMC cylinder. The co-polar  $(E_z^{s,0}, H_\phi^{s,0})$  and the cross polar  $(E_\phi^{s,0}, H_z^{s,0})$  components of the scattered field incident on the interface produces transmitted co-polar  $(E_z^{t,0}, H_\phi^{t,0})$  and cross polar  $(E_\phi^{t,0}, H_z^{t,0})$  field components into free space, and reflected co-polar  $(E_z^{r,0}, H_\phi^{r,0})$  and cross polar  $(E_\phi^{r,0}, H_z^{r,0})$  field components back into the dielectric half-space as second order field components, which are incident on the cylinder. These can be written as

$$E_z^{t,0} = \sum_{n=-\infty}^{\infty} D_n^0 H_n^{(2)}(k_0 \rho) e^{jn\phi} \quad (17)$$

$$H_\phi^{t,0} = \frac{1}{j\eta_0} \sum_{n=-\infty}^{\infty} D_n^0 H_n^{(2)'}(k_0 \rho) e^{jn\phi} \quad (18)$$

$$H_z^{t,0} = \frac{1}{j\eta_0} \sum_{n=-\infty}^{\infty} F_n^0 H_n^{(2)}(k_0 \rho) e^{jn\phi} \quad (19)$$

$$E_\phi^{t,0} = \sum_{n=-\infty}^{\infty} F_n^0 H_n^{(2)'}(k_0 \rho) e^{jn\phi} \quad (20)$$

$$E_z^{r,0} = \sum_{n=-\infty}^{\infty} G_n^0 H_n^{(2)}(k_1 \rho_{imag}) e^{jn\phi_{imag}} \quad (21)$$

$$H_\phi^{r,0} = \frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} G_n^0 H_n^{(2)'}(k_1 \rho_{imag}) e^{jn\phi_{imag}} \quad (22)$$

$$H_z^{r,0} = \frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} Q_n^0 H_n^{(2)}(k_1 \rho_{imag}) e^{jn\phi_{imag}} \quad (23)$$

$$E_\phi^{r,0} = \sum_{n=-\infty}^{\infty} Q_n^0 H_n^{(2)'}(k_1 \rho_{imag}) e^{jn\phi_{imag}} \quad (24)$$

where  $\rho_{imag}$  and  $\phi_{imag}$  are the circular cylindrical coordinates of the image cylinder shown in Fig. 1,  $D_n^0$  and  $G_n^0$  are the unknown co-polar transmitted and reflected field coefficients, respectively, while  $F_n^0$  and  $Q_n^0$  are the corresponding unknown cross polar field coefficients. Imposing the electric and magnetic field boundary conditions at the point P

on the planar interface (in Fig.1) corresponding to  $\rho=\rho_{imag}=d$ ,  $\phi=\pi$ , and  $\phi_{imag}=0$ , given by

$$E_z^{s,0} + E_z^{r,0} = E_z^{t,0} \quad (25)$$

$$H_\phi^{s,0} + H_\phi^{r,0} = H_\phi^{t,0} \quad (26)$$

$$E_\phi^{s,0} + E_\phi^{r,0} = E_\phi^{t,0} \quad (27)$$

$$H_z^{s,0} + H_z^{r,0} = H_z^{t,0} \quad (28)$$

yields the unknown expansion coefficients.

Substituting from (11)–(14) and (17)–(24) in (25) to (28), and considering the orthogonality of the exponential functions, we get

$$B_n^0 H_n^{(2)}(k_1 d) e^{jn\pi} + G_n^0 H_n^{(2)}(k_1 d) = D_n^0 H_n^{(2)}(k_0 d) e^{jn\pi} \quad (29)$$

$$B_n^0 H_n^{(2)'}(k_1 d) e^{jn\pi} - G_n^0 H_n^{(2)'}(k_1 d) = \frac{\eta_1}{\eta_0} D_n^0 H_n^{(2)'}(k_0 d) e^{jn\pi} \quad (30)$$

$$C_n^0 H_n^{(2)'}(k_1 d) e^{jn\pi} + Q_n^0 H_n^{(2)'}(k_1 d) = F_n^0 H_n^{(2)'}(k_0 d) e^{jn\pi} \quad (31)$$

$$C_n^0 H_n^{(2)}(k_1 d) e^{jn\pi} - Q_n^0 H_n^{(2)}(k_1 d) = \frac{\eta_1}{\eta_0} F_n^0 H_n^{(2)}(k_0 d) e^{jn\pi} \quad (32)$$

Using (29)–(32), the unknown coefficients can be obtained as

$$G_n^0 = \left( \frac{2 H_n^{(2)}(k_0 d)}{x_n H_n^{(2)}(k_1 d)} - 1 \right) B_n^0 e^{jn\pi} \quad (33)$$

$$D_n^0 = \frac{2 B_n^0}{x_n} \quad (34)$$

with

$$x_n = \left( \frac{\eta_1 H_n^{(2)'}(k_0 d)}{\eta_0 H_n^{(2)'}(k_1 d)} + \frac{H_n^{(2)}(k_0 d)}{H_n^{(2)}(k_1 d)} \right) \quad (35)$$

and

$$Q_n^0 = \left( \frac{2 H_n^{(2)'}(k_0 d)}{y_n H_n^{(2)'}(k_1 d)} - 1 \right) C_n^0 e^{jn\pi} \quad (36)$$

$$F_n^0 = \frac{2 C_n^0}{y_n} \quad (37)$$

with

$$y_n = \left( \frac{\eta_1 H_n^{(2)}(k_0 d)}{\eta_0 H_n^{(2)}(k_1 d)} + \frac{H_n^{(2)'}(k_0 d)}{H_n^{(2)'}(k_1 d)} \right) \quad (38)$$

The reflected fields that correspond to components in (21)–(24), upon arriving at the buried cylinder, act as secondary incident fields and create higher order scattered fields.

Using Graf's addition theorem, the reflected field components which act as secondary incident field components can be written as

$$E_z^{r,0} = \sum_{n=-\infty}^{\infty} \bar{G}_n^0 J_n(k_1 \rho) e^{jn\phi} \quad (39)$$

$$E_\phi^{r,0} = \sum_{n=-\infty}^{\infty} \bar{Q}_n^0 J_n'(k_1 \rho) e^{jn\phi} \quad (40)$$

where

$$\bar{G}_n^0 = \sum_{m=-\infty}^{\infty} G_m^0 H_{m-n}^{(2)}(2k_1 d) \quad (41)$$

$$\bar{Q}_n^0 = \sum_{m=-\infty}^{\infty} Q_m^0 H_{m-n}^{(2)}(2k_1 d) . \quad (42)$$

The corresponding magnetic field components can be obtained using Maxwell's equations.

The field components scattered by the cylinder in response to the above secondary field components incident on it, can be written referring to (11)–(14), by changing the superscript 0 to 1. The unknown scattered field expansion coefficients are evaluated by applying the PEMC boundary conditions on the surface of the cylinder, which can be written as

$$H_z^{r,0} + H_z^{s,1} + M (E_z^{r,0} + E_z^{s,1}) = 0 \quad (43)$$

$$H_\phi^{r,0} + H_\phi^{s,1} + M (E_\phi^{r,0} + E_\phi^{s,1}) = 0 . \quad (44)$$

Substituting for the above field components using their expansions and considering the orthogonality of the exponential functions, the obtained resulting equations can be written in a matrix form as

$$\begin{bmatrix} B_n^1 \\ C_n^1 \end{bmatrix} = \begin{bmatrix} \aleph_{11,n} & \aleph_{12,n} \\ \aleph_{21,n} & \aleph_{22,n} \end{bmatrix} \begin{bmatrix} \bar{G}_n^0 \\ \bar{Q}_n^0 \end{bmatrix} \quad (45)$$

where

$$\aleph_{11,n} = - \frac{M^2 \eta_1^2 J_n(k_1 a) H_n^{(2)'}(k_1 a) + J_n'(k_1 a) H_n^{(2)}(k_1 a)}{(1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} \quad (46)$$

$$\aleph_{12,n} = \frac{2M \eta_1}{\pi k_1 a (1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} \quad (47)$$

$$\aleph_{21,n} = - \frac{2M \eta_1}{\pi k_1 a (1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} \quad (48)$$

$$\aleph_{22,n} = - \frac{J_n(k_1 a) H_n^{(2)'}(k_1 a) + M^2 \eta_1^2 J_n'(k_1 a) H_n^{(2)}(k_1 a)}{(1 + M^2 \eta_1^2) H_n^{(2)}(k_1 a) H_n^{(2)'}(k_1 a)} . \quad (49)$$

Cylindrical wave expansions of the k-th order field components  $\Xi_\tau^{s,k}$  scattered by the cylinder, and the k-th order components  $\Xi_\tau^{r,k}$  and  $\Xi_\tau^{t,k}$  reflected and transmitted by the interface, respectively, for  $\Xi=E, H$  and  $\tau=z, \phi$  can be obtained by referring to (11)–(14), (21)–(24), and (17)–(20), respectively, and changing the superscript 0 in these equations to k.

The unknown expansion coefficients  $\Upsilon_n^k$  for  $\Upsilon=B, C, D, F, G, Q$  and for  $k=1,2,\dots$  can be obtained by starting with (45), then using (33), (34), (36), (37), (41), and (42), after changing the superscript 0 to 1, and then repeating the sequence of calculations.

The z and  $\phi$  components of the electric field in the free space region above the interface can then be written as

$$\tilde{L}_z = \sum_k E_z^{t,k} \quad (50)$$

$$\tilde{L}_\phi = \sum_k E_\phi^{t,k} \quad (51)$$

The co-polar and cross polar reflection coefficients are thus given by

$$\Gamma_{cp} = \frac{\tilde{L}_z}{E_z^i} \quad (52)$$

$$\Gamma_{xp} = \frac{\tilde{L}_\phi}{E_z^i} \quad (53)$$

### III. NUMERICAL RESULTS

Results are presented to show how the co-polar and cross polar reflection coefficient magnitudes at the point P in Fig. 1, vary with the size of the cylinder, for PEMC cylinders of different PEMC admittances and burial depths, as well as for when each cylinder with a given PEMC admittance is buried at different depths and in different types of dielectric media. For the sake of convenience, the normalized PEMC admittance has been expressed in the dimensionless form  $M\eta_1 = \tan \nu$ .

To validate the analysis and the software used for obtaining results,  $|\Gamma_{cp}|$  has been calculated at the point P in Fig. 1, for  $\nu=90^\circ$ ,  $\epsilon_r=2.0$ , and  $d=2.25$  cm, when the electrical radius  $k_0 a$  of the cylinder increases from 0.0 to 1.6. These results have been

compared in Fig. 2, with the corresponding results for a buried PEC circular cylinder considered in [10], and are in excellent agreement, verifying the accuracy of the analysis and the software used for obtaining the results.

Figure 3 shows the variation of  $|\Gamma_{cp}|$  and  $|\Gamma_{xp}|$  at P for a PEMC cylinder buried in a dielectric medium having a relative permittivity of 2.0, at a depth of 2.25 cm from the planar interface, for 5 different PEMC admittances, when the electrical radius of the cylinder varies from 0 to 1.6. In Fig. 3(a), the minima move towards the right as the value of  $\nu$  is increased and the sharpness of the first minimum also decreases with increasing  $\nu$ . In Fig. 3(b), we observe that the curves for  $\nu=15^\circ$  and  $\nu=75^\circ$  are essentially the same and so are those for  $\nu=30^\circ$  and  $\nu=60^\circ$ . Also, variations of  $|\Gamma_{cp}|$  with  $k_0a$  are quite rapid as compared to those of  $|\Gamma_{xp}|$ .

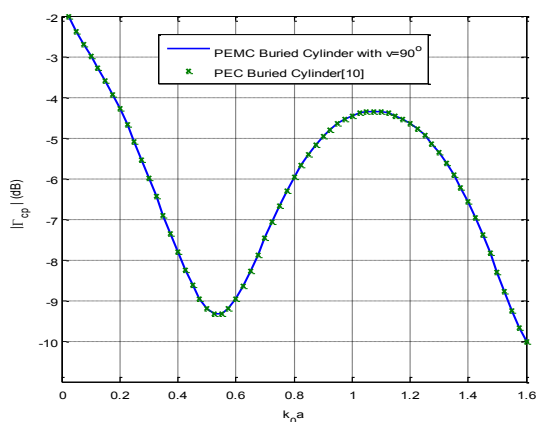


Fig. 2. Variation of the magnitude of co-polarized reflection coefficient versus  $k_0a$  for buried PEMC circular cylinders of radius  $a$ , with  $\nu=90^\circ$ ,  $\epsilon_r=2.0$ , and  $d=2.25$  cm.

Figure 4 shows the variations of  $|\Gamma_{cp}|$  and  $|\Gamma_{xp}|$  at P for the PEMC cylinders of Fig. 3, buried at a depth of 5 cm from the planar interface.  $|\Gamma_{cp}|$  in Fig. 4(a) are more oscillatory when compared with the same in Fig. 3(a). However, the variations of  $|\Gamma_{xp}|$  in Fig. 4(b) are not too different to those in Fig. 3(b). Yet, the values of  $|\Gamma_{cp}|$  and  $|\Gamma_{xp}|$  in Fig. 4 are smaller, when compared with the corresponding values in Fig. 3.

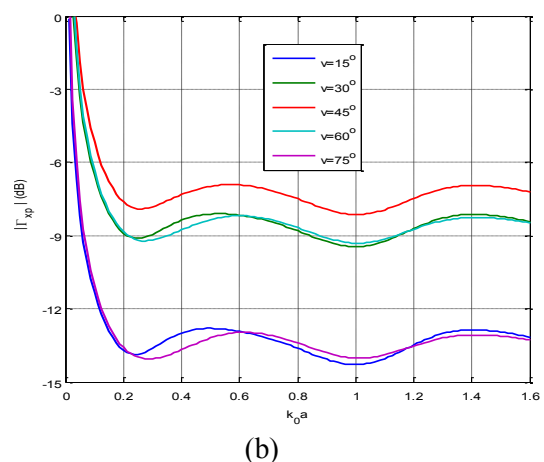
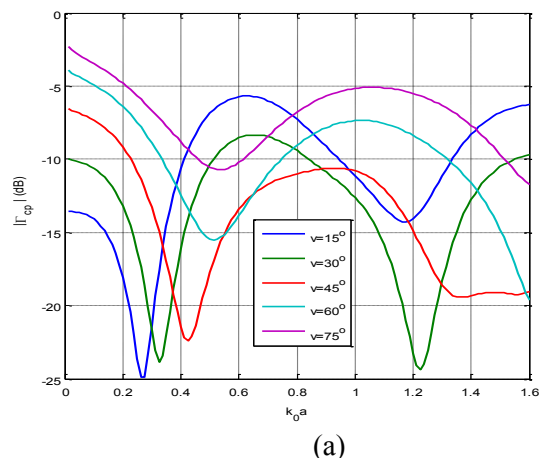
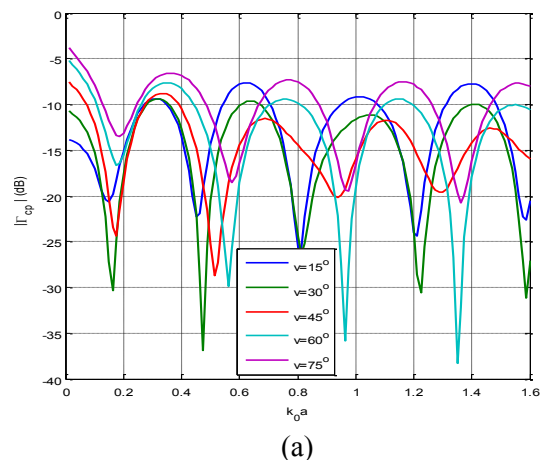
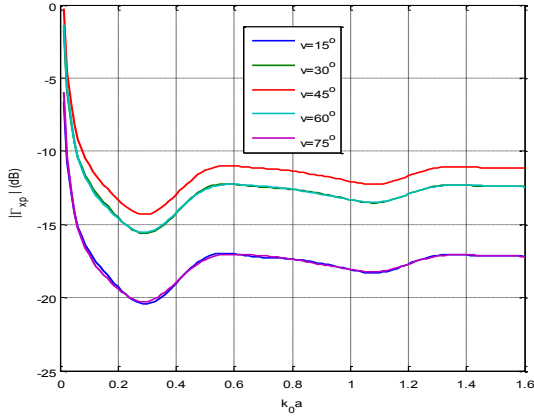


Fig. 3. Variations of the magnitudes of reflection coefficients versus  $k_0a$  for buried PEMC circular cylinders of radius  $a$ , with  $\epsilon_r=2.0$ ,  $d=2.25$  cm, for different values of  $\nu$ ; (a) co-polar coefficient; (b) cross polar coefficient.





(b)

Fig. 4. Variations of the magnitudes of reflection coefficients versus  $k_0a$  for buried PEMC circular cylinders of radius  $a$ , with  $\epsilon_r = 2.0$ ,  $d = 5.0$  cm, for different values of  $\nu$ ; (a) co-polar coefficient; (b) cross polar coefficient.

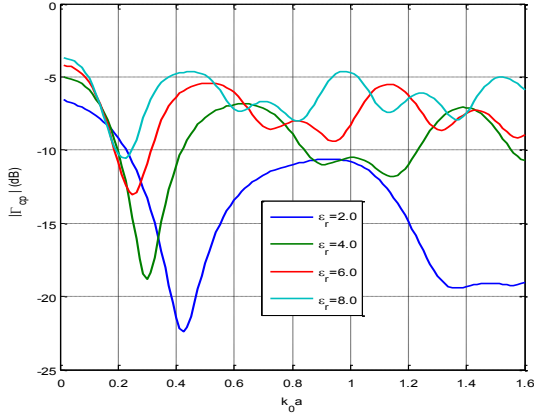


Fig. 5. Variations of the magnitudes of co-polar reflection coefficients versus  $k_0a$  for buried PEMC circular cylinders of radius  $a$ , with  $d = 2.25$  cm,  $\nu = 45^\circ$ , and for different  $\epsilon_r$ .

Variations of  $|\Gamma_{cp}|$  against the electrical radius  $k_0a$  of each buried PEMC cylinder, for  $d = 2.25$  cm,  $\nu = 45^\circ$ , and for different  $\epsilon_r$  are shown in Fig. 5. As  $\epsilon_r$  increases from 2.0 to 8.0, the oscillations in the plots also increase. The value of  $|\Gamma_{cp}|$  at a given  $k_0a$  is in general high for a high value of  $\epsilon_r$ .

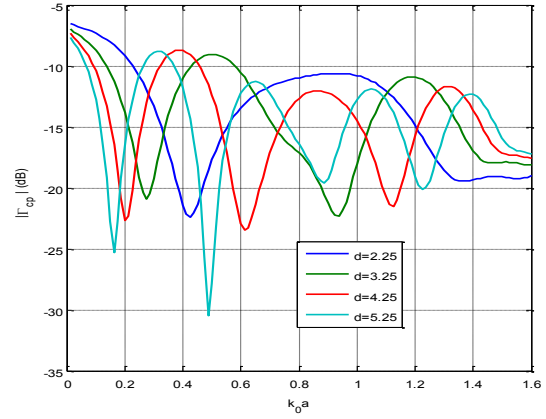


Fig. 6. Variations of the magnitudes of co-polar reflection coefficients versus  $k_0a$  for buried PEMC circular cylinders of radius  $a$ , with  $\epsilon_r = 2.0$ ,  $\nu = 45^\circ$ , and for different values of the burial depth  $d$ .

Variations of  $|\Gamma_{cp}|$  against the electrical radius  $k_0a$  of each buried PEMC cylinder, for  $\nu = 45^\circ$ ,  $\epsilon_r = 2$ , and for different burial depths  $d$  are shown in Fig. 6. As the value of  $d$  increases from 2.25 to 6.25, the oscillations in the plots also increase. Also, the corresponding minima and maxima move to the left as  $d$  increases.

#### IV. CONCLUSION

A rigorous solution that does not require inversion of matrices and based on the multiple reflection of waves has been obtained for scattering by a PEMC cylinder buried in a dielectric half-space, when it is illuminated by a plane wave propagating in free space incident normally on the interface separating the two media. Numerical results have been given in the form of co-polar and cross polar free space reflection coefficient magnitudes, to exhibit how they vary with the size, burial depth, and PEMC admittance of the buried cylinder, as well as with the permittivity of the burial medium. Results for PEC and PMC cylinders can be obtained from this analysis, as special cases. The solution process can also be used for solving related inverse scattering problems.

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