

# Metal Mesh Metasurfaces as Dual-Band Bandpass Filters for Terahertz Frequencies

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**ABSTRACT** In this paper, we propose a new design strategy for metal mesh filters (MMFs) based on the analysis of the spatial symmetry of bound states in the continuum (BICs), and manipulation and control of their characteristics using resonances that originate from the BICs when the structure of MMF is spatially perturbed. The design of a dual-band polarization-insensitive terahertz bandpass filter with wide upper stopband characteristics using a single conducting layer patterned with rectangular holes is presented. The transmission response of the MMF with two poles is obtained to realize dual-band characteristics and three zeros to suppress the stopband. The proposed design has achieved broadband transmission characteristics for both TE and TM polarizations, with center frequencies at 0.516THz and 0.734THz, 3dB bandwidths of 25% and 17%, respectively, and upper stopband from 0.887THz to 1.6THz with over 10dB suppression.

**INDEX TERMS** Bound states in the continuum, dual-band, filters, terahertz.

## I. INTRODUCTION

**R**APIDLY expanding terahertz (THz) technology has various potential applications in medicine, space, security, communications, and environmental sensing [1–4]. Thus, research into active and passive THz devices with specific characteristics, such as multiband operation, high sensitivity, wideband operation, etc., is particularly important for THz technology applications [5]. There is an increasing demand for THz functional devices such as waveguides, polarizers, modulators, antennas, sensors, and bandstop/bandpass filters [6–11].

THz bandpass filters are one of the important components used to transmit desired frequencies with better selectivity and reject others. They are usually composed of periodically arranged unit cells, which are either conducting patches printed on a dielectric substrate or slots/apertures etched out of a conducting plate [12].

To meet required characteristics such as single- or multi-band, wide-band or high Q selectivity, sharp rejection, etc. various designs have been proposed [12–18]. They can be grouped into several types. The first type is filters based on planar structures composed of metal patches [13,14], metal apertures [15–20] or complimentary elements patterned on a dielectric substrate [21]. Depending on the design specifications, single or multilayer structures with elements of various shapes (crossed dipoles, circular or rectangular rings, split rings, etc.) are used. Better frequency selectivity for structures of this type can usually be obtained with a multilayer design, which is more complicated and can be bulky. The presence of dielectrics in such structure additionally leads to non-radiative loss. Another type of

filters is based on structures with 3D elements [22]. They can have the same frequency response as multilayer filters of the previous type, but are relatively complex to manufacture, compared to traditional cascades of planar metal elements and dielectric layers.

The third type is metal mesh filters, which are conductive surfaces periodically patterned with holes [23]. They are compact, can exhibit excellent transmission characteristics, and have the advantage of a simple manufacturing process and better frequency tuning possibilities compared to other types of filters. This type of bandpass filters exhibits high transmission at the central frequency, adjustable bandwidth, and good rejection of sideband frequencies [23]. For ease of manufacture, mesh filters are usually patterned with elements of a simple shape such as rectangular [18–20], circular [15,17], or crossed holes [23].

To obtain a dual band response, one or a combination of the following techniques is applied when designing MMFs: layered or stacked structures, perturbation of a single-layer structure, or unit cells with multi-resonant elements [16–19]. For single layer structures, resonant responses are caused by aperture resonances [24]. Two or more holes of different sizes located in a unit cell form independent resonances with very weak coupling between adjacent ones, and frequency selectivity characteristics can be achieved in both the lower and upper bands by tuning the sizes adjusting holes. However, by manipulating the electrical lengths of the elements, one can easily control only the resonant frequencies, but not the bandwidths or the level of out-of-band rejection. For multilayer structures, the

resonant responses are caused by the Fabry-Perot resonances, which depend on the thickness of the structures. As a result, the control of their characteristics is determined by the peculiarities of the resonances. In this case, selectivity and stopband suppression can be better than for single-layer structures. However, the corresponding designs can be more complex and bulky, which can result in additional loss.

In this paper, we propose a new strategy to design dual-band bandpass MMFs based on the analysis of spatial symmetry of bound states in the continuum (BICs) and manipulation and control of the resonances that originate from the BICs when the structure of MMF is spatially perturbed.

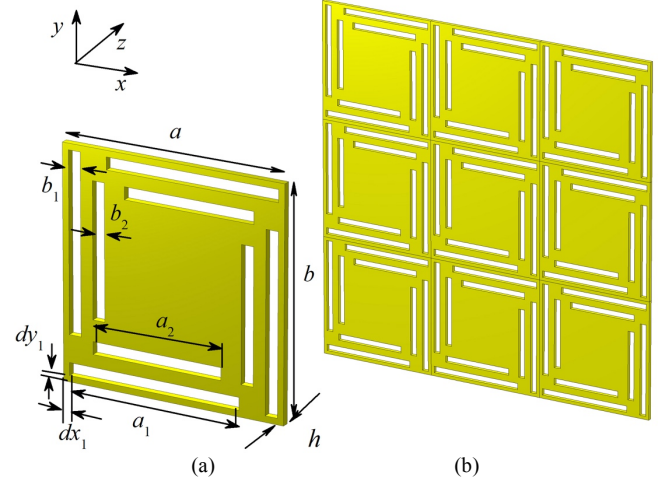
BICs are infinite-lifetime eigenmodes that coexist with a continuous spectrum of radiating waves but remain completely confined without any radiation. Its existence was first predicted by von Neumann and Wigner in 1929 [25]. Recently, extensive studies have been conducted on BICs in various periodic structures, motivated by their potential applications in resonant enhancement, lasing, and filtering [26-29].

Most studies have focused on high-Q resonances in the structures. Meanwhile, the BICs theory can be successfully applied to realize resonances with different Q-factors. If one understands spatial distributions of the BICs in a periodic cell, it allows the manipulation of poles and zeros by adapting the cell composition, thereby improving the frequency selectivity of the passband and effectively suppressing stopbands. By preserving a certain type of supercell symmetry, it is possible to control the transformation of the required BICs into QBICs (quasi-bound states in the continuum) while keeping the others symmetry-protected. In this case, lattice resonances determine the resonant response. Strong coupling effects between elements of the supercell are observed, and resonant frequencies no longer depend on the electrical lengths of the elements. Meanwhile, there are more degrees of freedom to control resonances. Manipulating with the element coupling strength, one can simultaneously control resonant frequencies and bandwidths. Moreover, because of non-orthogonality of the BICs, the incident wave can couple not only the lower order BICs forming passband response but also to the higher order ones suppressing stopband with zeros due to coupling effects between them.

## II. THE PROBLEM FORMULATION AND SOLUTION METHOD

To design the filter, we consider a metasurface consisting of a thin conductive metal screen of thickness  $h$  periodically patterned with holes arranged in unit cells with periods  $a$  and  $b$  along the  $x$ - and  $y$ -axis, respectively (Figure 1). To solve the scattering problem, the metal elements are modeled as infinitesimally thin with impedance  $\xi = i\sqrt{\mu_0/(4\epsilon_0\epsilon_m)} \cdot \text{ctg}(h\pi f \sqrt{\epsilon_0\epsilon_m\mu_0})$ ,

where  $\epsilon_0$  and  $\mu_0$  are the permittivity and permeability of free space, respectively,  $\epsilon_m = 1 + i\sigma_m/(2\pi f\epsilon_0)$  is the relative permittivity of the metal, and  $\sigma_m$  is the conductivity of the metal. The most efficient method for solving the plane wave scattering problem on the structure can be obtained by reducing the computation domain to the holes in the unit cell.



**FIGURE 1.** Proposed THz MMF. (a) Geometry of the periodic MMF cell. (b) Perspective view of the proposed MMF.

The electromagnetic fields  $(E_0, H_0)$  and  $(E_1, H_1)$ , above and below the metasurface, respectively, must satisfy the Floquet theorem so that they can be expanded into a complete set of Floquet modal basis functions with unknown complex amplitudes.

The transverse electromagnetic fields with respect to the screen plane  $S = \{\vec{r} : z = 0\}$  can be presented as follows:

$$\vec{E}_{t,j} = \sum_{p=-\infty}^{\infty} (A_{j,p}^+ e^{i\gamma_p z} + A_{j,p}^- e^{-i\gamma_p z}) \sqrt{w_p} e_p, \quad (1)$$

$$H_{t,j} = \sum_{p=-\infty}^{\infty} (A_{j,p}^+ e^{i\gamma_p z} - A_{j,p}^- e^{-i\gamma_p z}) \frac{z \times e_p}{\sqrt{w_p}} \quad (2)$$

where  $A_{0,p}^-$ ,  $A_{1,p}^+$  are the incident amplitudes, and  $A_{0,p}^+$ ,  $A_{1,p}^-$  are the unknown scattered ones. Each term  $e_p$  in the series (1) defines the transverse electric field of TE mode  $e_p = e_{mn}^{(h)}$  or TM mode  $e_p = e_{mn}^{(e)}$ :

$$e_{m,n}^{(e)} = \frac{\nabla_t e^{ik_{x,m}x + ik_{y,n}y}}{k_{mn} \sqrt{ab}}, \quad e_{m,n}^{(h)} = \frac{-z \times \nabla_t e^{ik_{x,m}x + ik_{y,n}y}}{k_{mn} \sqrt{ab}} \quad (3)$$

where  $\nabla_t$  denotes the 2-D gradient operating on the coordinates of the transverse plane  $xOy$ ;  $\gamma_p = \sqrt{k^2 - k_p^2}$ ,

$w_p^{(e)} = w\gamma_p/k$ ,  $w_p^{(h)} = kw/\gamma_p$  are the propagation constants and mode impedances;  $k_{x,m} = k_x^{(i)} + 2\pi m/a$ ,  $k_{y,n} = k_y^{(i)} + 2\pi n/b$ ,  $k_p = k_{mn} = \sqrt{k_{x,m}^2 + k_{y,n}^2}$  are the transverse wave numbers;  $k$  and  $w$  are wave number and impedance of free space;  $\bar{k}^{(i)} = (k_x^{(i)}, k_y^{(i)})$  is the incident wave propagation vector.

To solve the problem, it is necessary to match the electric and magnetic fields on both sides of the boundaries  $S = S^{(m)} \cup S^{(a)}$ , satisfying the following boundary conditions:

$$(E_{t,0} - E_{t,1})|_S = 0, \quad (4)$$

$$z \times (H_{t,0} - H_{t,1})|_{S^{(a)}} = 0, \quad (5)$$

$$z \times (H_{t,0} - H_{t,1})|_{S^{(m)}} = \frac{1}{2\xi} (E_{t,0} + E_{t,1})|_{S^{(m)}} \quad (6)$$

where  $S^{(a)}$  and  $S^{(m)}$  correspond to the surfaces of holes and metal, respectively.

Applying condition (4) of the electric field continuity across  $S$  and taking into account the orthogonality of the  $e_p$  set, the equations relating the mode amplitudes in half-spaces above and below the screen with electric field  $M = E_{t,0} = E_{t,1}$  on the boundary  $S$  are obtained

$$A_{j,p}^+ + A_{j,p}^- = \frac{1}{\sqrt{w_p}} \int_S (M \cdot e_p^*) dS, \quad (7)$$

where the symbol  $*$  denotes the complex conjugate.

Matching the magnetic fields according to conditions (5), (6) and substituting (7) to exclude  $A_{0,p}^+$ ,  $A_{1,p}^-$ , the equations relating  $M$  on  $S^{(a)}$  and  $S^{(m)}$  with amplitudes of the incident field are obtained

$$\frac{M}{\xi} = \sum_p \left( \frac{2}{\sqrt{w_p}} (A_{0,p}^- + A_{1,p}^+) - \frac{2}{w_p} \int_S (M \cdot e_p^*) dS \right) e_p|_{S^{(m)}} \quad (8)$$

$$0 = \sum_p \left( \frac{2}{\sqrt{w_p}} (A_{0,p}^- + A_{1,p}^+) - \frac{2}{w_p} \int_S (M \cdot e_p^*) dS \right) e_p|_{S^{(a)}} \quad (9)$$

Equations (7)-(9), which express unknown amplitudes  $A_{0,p}^+$ ,  $A_{1,p}^-$  in terms of  $M$  on the boundary  $S$  can be simplified by excluding  $M = M_m$  on the  $S^{(m)}$ . Therefore,  $A_{0,p}^+$ ,  $A_{1,p}^-$  can be expressed in terms of  $M = M_a$  on the  $S^{(a)}$  only. For this sake, the relation between  $M_m$  and  $M_a$  can be obtained from (8)

$$\begin{aligned} \int_{S^{(m)}} \left( \frac{1}{\xi} + \frac{2}{w_p} \right) (M_m \cdot e_p^*) dS = \\ = \frac{2}{\sqrt{w_p}} (A_{0,p}^- + A_{1,p}^+) - \frac{2}{w_p} \int_{S^{(a)}} (M_a \cdot e_p^*) dS \end{aligned} \quad (10)$$

followed by exclusion  $M_m$  using (9).

The result is the Fredholm integral equation of the first kind with respect to  $M_a$ , which is defined over the cross sections of the holes  $S^{(a)}$

$$\begin{aligned} \int_{S^{(a)}} \sum_p \frac{1}{w_p/2 + \xi} (M_a \cdot e_p^*) dS e_p|_{S^{(a)}} = \\ = \sum_p \frac{\sqrt{w_p}}{w_p/2 + \xi} (A_{0,p}^- + A_{1,p}^+) e_p|_{S^{(a)}} \end{aligned} \quad (11)$$

Unknown amplitudes of the modes scattered on the screen can be obtained using the following formulas

$$\begin{aligned} A_{0,p}^+ &= -A_{0,p}^- + \frac{\xi}{w_p/2 + \xi} (A_{0,p}^- + A_{1,p}^+) + \\ &+ \frac{\sqrt{w_p}/2}{w_p/2 + \xi} \int_{S^{(a)}} (M_a \cdot e_p^*) dS \\ A_{1,p}^- &= -A_{1,p}^+ + \frac{\xi}{w_p/2 + \xi} (A_{0,p}^- + A_{1,p}^+) + \\ &+ \frac{\sqrt{w_p}/2}{w_p/2 + \xi} \int_{S^{(a)}} (M_a \cdot e_p^*) dS \end{aligned} \quad (12)$$

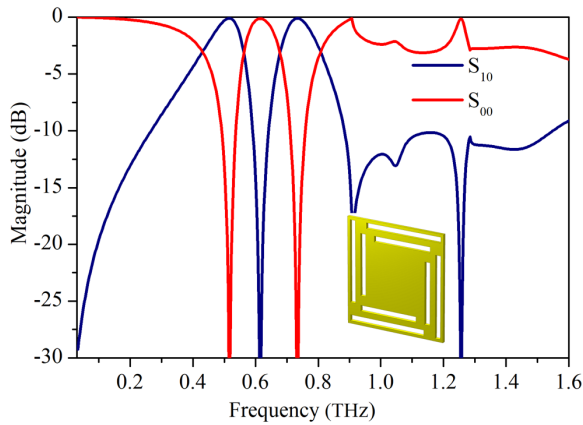
### III. PROPOSED FILTER DESIGN

The designed MMF structure is a periodic array of compound supercells with rectangular holes in conducting metal of thickness  $h$ . It contains two sets ( $i=1,2$ ) of four identical rectangular holes of length  $a_i$  and width  $b_i$  located in such a way that the supercell has a four-fold rotational symmetry (it is spatially invariant in the  $xy$ -plane for rotations through angles  $\alpha = \pi n/2$ ,  $n=1,2,3$ ). The hole positions depend on two parameters  $dx_i$  and  $dy_i$ , which are the distances of the holes from the supercell boundaries.

The final geometry dimensions are given as follows:  $a = b = 330 \mu m$ ,  $a_1 = 255 \mu m$ ,  $a_2 = 193 \mu m$ ,  $b_1 = b_2 = 18 \mu m$ ,  $dx_1 = dy_1 = 9 \mu m$ ,  $dx_2 = dy_2 = 45 \mu m$ , and  $h = 10 \mu m$ . The conductive metal is aluminum with frequency independent conductivity  $\sigma_{Al} = 3.56 \cdot 10^7 S/m$ .

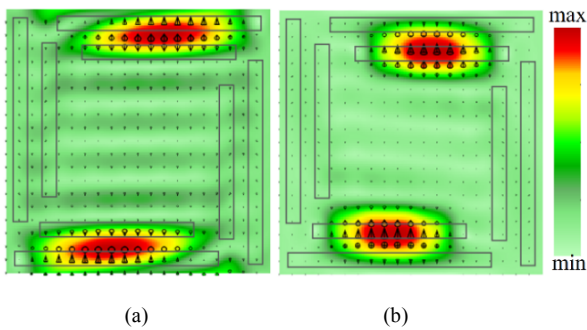
A plane wave scattering problem, reduced to the equations (11) and (12), is solved using the full-wave moment method. Figure 2 shows the simulated transmission and reflection values for the MMF. A dual-band transmission response is observed with resonances at 0.516THz and 0.734THz. For the first band, the maximum transmission is 99% with the 3dB relative bandwidth of 25%, and for the second band, the maximum transmission

is 98% with the 3dB relative bandwidth of 17%. The upper stopband has three zeros and suppression over 10dB from 0.887THz to 1.6THz.



**FIGURE 2.** Simulated magnitudes of the MMF transmission  $S_{10}$  and reflection  $S_{00}$  responses at normal incidence. Two transmission resonances are found at 0.516THz, and 0.734THz. The inset shows the periodic MMF cell structure.

The resonances in the frequency response are caused by the excitation of four eigenmodes belonging to two orthogonal sets with the eigenfrequencies  $f_1 = 0.548 - i \cdot 0.069 \text{ THz}$  and  $f_2 = 0.7301 - i \cdot 0.0796 \text{ THz}$ . The eigenmodes of each set can only couple to TE or TM plane wave, resulting in two resonances in the frequency response when one of the waves is incident on the MMF. The electric and magnetic fields of eigenmodes for these two sets are completely identical to each other, but have orthogonal distributions along the x-axis and the y-axis.



**FIGURE 3.** Electric field distribution at the resonant frequencies in the case of TE plane wave incidence. The cones designate a value and direction of the electric field vectors. (a) 0.516THz. (b) 0.734THz.

Figure 3 shows the electric field distribution at the resonant frequencies 0.516THz and 0.734THz. The resonance at 0.734THz is an aperture resonance (Figure 3(b)). The electric field is mainly concentrated in smaller holes, and electric coupling between adjacent holes is very weak. On the contrary, for the electric field at 0.516THz, strong coupling between adjacent holes is observed, while

the electric field is concentrated not inside the holes, but between them (Figure 3(a)). It is a lattice resonance [30,31].

The resonance is due to coupling incident wave with the eigenmode, electric field of which is formed not by individual elements of the cell, but by the entire cell, and it is determined by spatial symmetry of the cell. Such resonances appear when the BIC is tuned to QBIC due to perturbation in supercell structure.

#### IV. FILTER DESIGN STRATEGY AND DISCUSSION

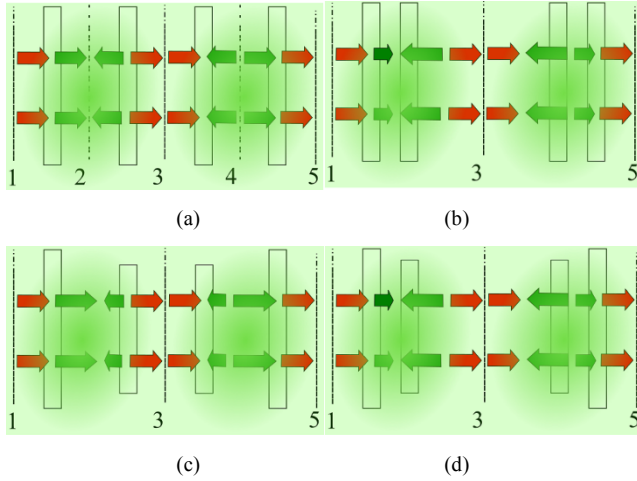
In this section, we consider the main stages of filter synthesis. The synthesis procedure is based not on analyzing the properties of individual elements forming a periodic supercell, as is usually done, but on considering the entire supercell as a scatterer that forms the scattering characteristics due to its spatial symmetry. To obtain the required characteristics, eigenmodes of the metasurface are considered, and their contribution to the scattered field is analyzed when spatial perturbations are introduced into the metasurface supercell structure.

As is known, eigenmodes, which are nontrivial solutions of source-free Maxwell's equations, contribute to the scattered field when the structure is excited by an external source, leading to appearance of resonances. However, there are also eigenmodes that do not contribute to the scattered field due to various protection mechanisms such as spatial symmetry, accidental parameter tuning, etc. These eigenmodes are known as BICs. They are currently being actively investigated for narrowband device applications. However, the theory of these eigenmodes can be effectively applied to the synthesis of various filters.

We start with a periodic structure arranged as an array of supercells containing four rectangular holes equidistantly distributed along the  $x$ -axis (Figure 4(a)). The periodic supercell has mirror symmetry planes along the  $x$ -axis, passing through the centers of the holes and the middle planes between them, and, as follows from the theory of symmetry, such a structure has eigenmodes with symmetric (magnetic walls) and antisymmetric (electric walls) distributions of electromagnetic fields relative to the mirror symmetry planes [32]. In the case of excitation of the structure by a normally incident TM plane wave, eigenmodes with antisymmetric distributions inside the supercell can only be coupled to the incident wave and contribute to the scattering field.

In the frequency band where only one wave propagates, an eigenmode with electric walls in all mirror symmetry planes can only be coupled to the incident field, of resulting in a frequency response with a single transmission resonance. Other eigenmodes are symmetry protected BICs, which can be excited only if perturbations are introduced into the supercell structure. Exciting the BICs can enable additional transmission resonances, improve passband frequency selectivity, and control suppression in stopband. For this case, there are six BICs [32], which can be

transformed into QBICs by various perturbations in the supercell.



**FIGURE 4.** Schematic illustration of the electric field distribution of the eigenmode when various perturbations are introduced into the supercell. The symmetrical distribution of the electric vectors projected onto the  $xy$ -plane is shown by the arrows. Two equally sized green and red arrows designate magnetic and electric walls, respectively.

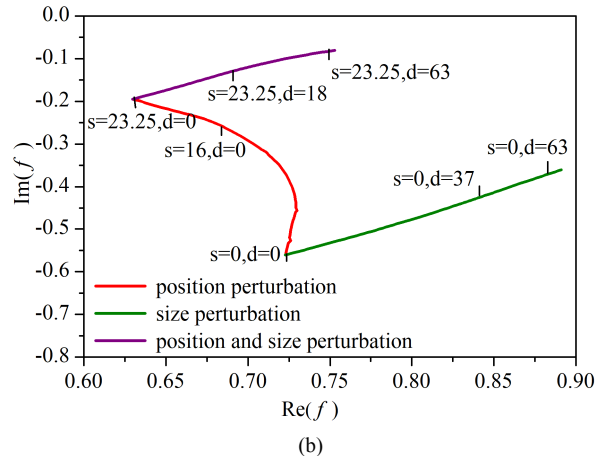
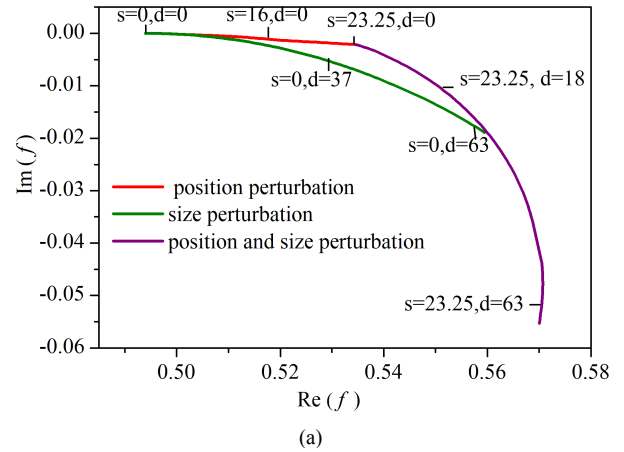
For our design, we consider one of them. A sketch of the electric field distribution of which is shown in Figure 4(a). The electric field of the BIC has three electric walls in planes 1, 3 and 5 (Figure 4(a)), and two magnetic walls in planes 2 and 4. The presence of magnetic walls in the electric field distribution of the BIC prevents its coupling to the incident TM plane wave. Thus, in order to excite the eigenmode, it is necessary to introduce perturbations into the cell structure that break the mirror symmetry in the planes 2 and 4 and thereby destroy the magnetic walls in the eigenmode electric field distribution. In this case, the symmetry must be preserved in planes 1, 3 and 5 to avoid excitation of other eigenmodes, that lead to undesired resonances in both passband and stopband. This can be achieved in several ways: by symmetrically shifting the holes relative to the supercell middle plane (Figure 4(b)), by changing the size of the internal holes inside the supercell (Figure 4(c)), and by both symmetrically shifting the holes and changing the size of the internal holes (Figure 4(d)).

To analyze dependence of the transmission response for each case, two parameters  $s$  and  $d$  are introduced describing perturbations inside the periodic supercell. The parameter  $s$  describes perturbations of the position of the holes inside the supercell  $\Delta x_1 = a/8 - b_1/2 - s$  and  $\Delta x_2 = 3a/8 - b_2/2 - 3s$ , and the parameter  $d = a_1 - a_2$  describes perturbation of the sizes of the holes. The case  $s = 0$  and  $d = 0$  corresponds to an unperturbed supercell (Figure 4(a)).

Figure 5 shows the curves describing the movement of the eigenfrequencies  $f_1$  and  $f_2$  on the first sheet of the Riemann surface with changing supercell parameters  $s$  and

$d$ . The corresponding eigenmodes provide a dual-band response for the MMF. The values of  $s$  and  $d$  corresponding to eigenfrequency values are marked on the curves.

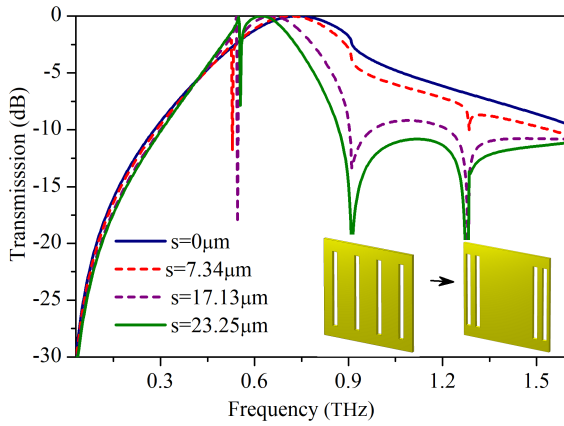
When perturbation is introduced in a supercell by changing the position of the holes (the parameter  $s$  takes values from  $s = 0 \mu\text{m}$  to  $s = 23.25 \mu\text{m}$ ,  $d = 0$ ), the BIC (red curve in Figure 5(a)) tuned to the QBIC, and its eigenfrequency  $f_1$  starts moving in the complex plane from the real value  $f_1 = 0.494 \text{ THz}$  to the value  $f_1 = 0.535 - i0.002 \text{ THz}$ . The imaginary part of  $f_1$  is weakly dependent on changes in  $s$ , while the real part increases relatively quickly. The eigenfrequency  $f_2$  changes from  $f_2 = 0.723 - i0.56 \text{ THz}$  to  $f_2 = 0.63 - i0.19 \text{ THz}$ . Thus the motions of  $f_1$  and  $f_2$  are opposite to each other in the complex plane.



**FIGURE 5.** The motions of eigenfrequencies on the first sheet of the Riemann surface with changing supercell parameters  $s$  and  $d$ : (a) eigenfrequency  $f_1$ ; (b) eigenfrequency  $f_2$ .

The frequency responses of the MMF corresponding to different values of  $s$  are shown in Figure 6 in the case of excitation by an incident TM-plane wave. As can be seen,

for any perturbed case ( $s \neq 0$ ), there are two resonances in the frequency response, in contrast to the unperturbed case ( $s = 0$ ), where only a single resonance is observed. However, the first resonance caused by the QBIC contribution to the scattered field in the case of perturbed geometry, is weakly expressed due to loss and the fact that the imaginary part of the QBIC eigenfrequency  $f_1$  is quite small. Its center frequency also cannot be controlled efficiently. Meanwhile, perturbations lead to the appearance of two transmission zeros and a significant suppression of the transmitted field in the upper stopband compared to the unperturbed case. As for the other resonance, observed for both perturbed and unperturbed cases, its center frequency and quality factor can be manipulated. A more perturbed geometry results in a lower resonant frequency and a higher quality factor for the resonance.

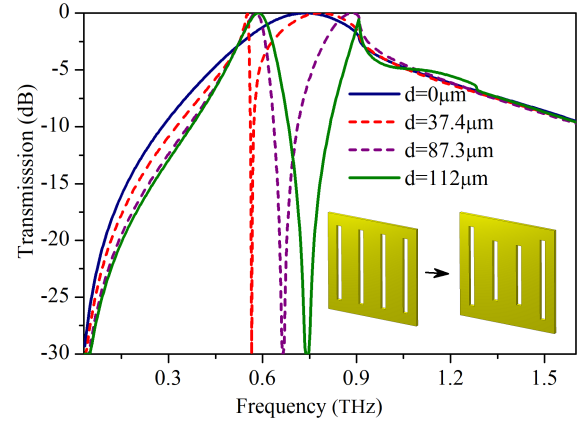


**FIGURE 6.** Dependence of the transmission response on the symmetrical shifts of holes inside the supercell. The inset shows the transformation of the supercell geometry from the unperturbed case with uniformly distributed holes to the case with two paired holes symmetrically located relative to the center plane of the supercell.

When supercell perturbations occur due to a change in the lengths of holes (the parameter  $d$  takes values from  $d = 0$  to  $d = 63.75 \mu\text{m}$ ,  $s = 0$ ), the eigenfrequency  $f_1$  (green curve in Figure 5(a)) moves in the complex plane from the real value  $f_1 = 0.494 \text{ THz}$  to the value  $f_1 = 0.56 - i0.2 \text{ THz}$ , while the value of eigenfrequency  $f_2$  is changed from  $f_2 = 0.723 - i0.56 \text{ THz}$  to  $f_2 = 0.89 - i0.36 \text{ THz}$ . In contrast to the previous type of perturbation, the imaginary parts of the eigenfrequencies exhibit opposite behavior here. As  $d$  increases, the imaginary part of  $f_1$  decreases, while the imaginary part of  $f_2$  increases.

The frequency responses of the MMF corresponding to different values of  $d$  are shown in Figure 7. As in the previous case, there are two resonances in the frequency response when perturbations are introduced in the cell geometry. In this case, it is possible to manipulate the quality factors of both resonances, but it is impossible to

obtain a frequency response with two resonances having commensurate quality factors. One of the resonances quickly moves to higher frequencies while its quality factor increases.



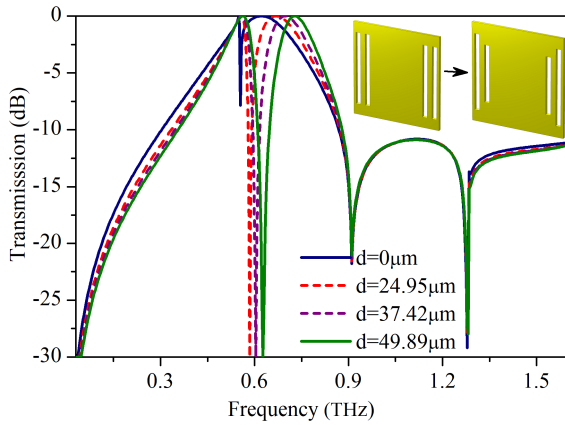
**FIGURE 7.** Dependence of the transmission response on the change in the length of hole. The inset shows the transformation of the supercell geometry from the unperturbed case to the case with two pairs of uniformly distributed holes of different lengths.

It should also be noted that in this case, transmission suppression is not observed in the upper stopband and the frequency response differs little for all values from  $d = 0$  to  $d = 63.75 \mu\text{m}$ .

Considering the opposite behavior of the real and imaginary parts of the eigenfrequencies  $f_1$  and  $f_2$  in the two cases, we can conclude that their values can be controlled by simultaneously changing the positions ( $s \neq 0$ ) and lengths ( $d \neq 0$ ) of the holes.

The motion of the eigenfrequencies in the complex plane, with parameter  $d$  varying from  $d = 0$  to  $d = 63.75 \mu\text{m}$  and the parameter  $s$  fixed at  $s = 23.25 \mu\text{m}$ , is shown in Figure 5 by the purple curves. As can be seen, both the imaginary and real parts of the eigenfrequencies can be effectively controlled to achieve the necessary positions in the complex plane, resulting in the required transmission characteristics. The frequency responses for various values of  $d$  in this case are shown in Figure 8. The two resonances observed in the figure can be controlled to obtain the desired characteristics. Changing lengths of holes leads to a decrease in the quality factor of the first resonance and an increase in the quality factor of the second resonance. The second resonance slowly moves to higher frequencies. In this way, the distance between the center resonant frequencies, as well as the quality factors, can be controlled to obtain the required bandwidths.

Additionally, the appearance of transmission zeros in the upper frequency range is observed, which is associated with the excitation of higher eigenmodes with eigen frequencies in this range and their coupling to the incident wave.



**FIGURE 8.** Dependence of the transmission response on changes in the lengths of symmetrically shifted holes in the supercell. The inset shows the transformation of the supercell geometry from the case with two pair of identical holes symmetrically located relative to the supercell center plane to the case with two pairs of holes of different lengths.

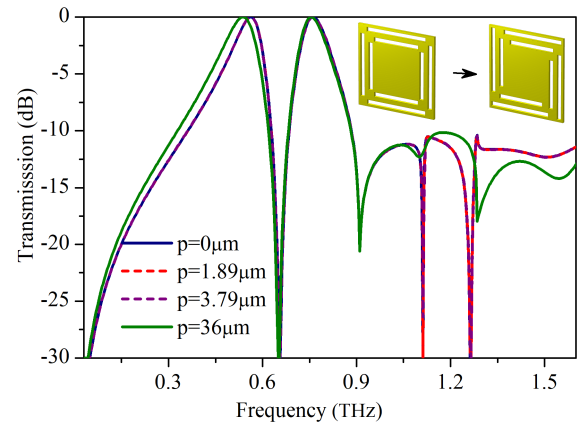
As can be seen in the inset of Figure 8, the supercell can be easily modified to achieve a polarization-independent response. To achieve this goal, the supercell must be complemented with holes that are orthogonal to those shown in the inset. The structure modified in this has two orthogonal sets of eigenmodes, which can couple to both incident TM and TE waves (Figure 9). In this case, the modified supercell has planes of mirror symmetry not only along the  $x$ -axis and  $y$ -axis, but also along the diagonals of the supercell. The latter do not influence the transmission response in the passband, however, they can introduce some changes in the stopband. At the same time, the stopband characteristics can be improved by transforming the mirror symmetry to  $C_4$  rotational symmetry of the supercell. To do this, we introduce the parameter  $p$ , which changes the distances between the holes along the  $y$ -axis  $\Delta y_i = (a - a_i)/2 + p$  and along the  $x$ -axis  $\Delta x_i = (a - a_i)/2 + p$ .

The value  $p = 0$  corresponds to the case of mirror symmetry, and the value  $p \neq 0$  corresponds to rotationally symmetrical geometry. In structures with  $n$ -fold rotational symmetry (for  $n > 2$ ), cross-polarization does not occur in the reflected field for the case of normally incident plane wave [33], but at the same time, such a transition makes it possible to improve the characteristics in the stopband by obtaining additional transmission zeros, as one can see in Figure 9.

## V. CONCLUSION

A new strategy for designing metal mesh filters (MMF) is proposed, based on the spatial symmetry analysis of bound states in the continuum (BICs). It involves manipulating and controlling resonances by transforming BICs into the resonances by spatial perturbations in the MMF structure. The design of a dual-band polarization insensitive terahertz

bandpass filter with wide upper stopband characteristics using a single conducting layer patterned with rectangular holes is presented. The transmission response of the MMF with two poles is obtained to realize dual-band characteristics and three zeros to suppress the stopband.



**FIGURE 9.** Dependence of the transmission response on shifts of the holes that break the mirror symmetry of the supercell. The inset shows the transformation supercell geometry from the mirror symmetrical case to rotationally symmetrical case.

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