

# Microwave Scattering Characteristics of a Cylindrical Conductor Coated by Dispersive Metamaterials with an Intervening Air Gap

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**ABSTRACT** Plane wave scattering characteristics in the microwave regime (0.1 GHz – 10 GHz), of a conducting cylinder coated by a layer of metamaterial (MTM) having dispersive and lossy constitutive parameters – permittivity ( $\epsilon$ ) and permeability ( $\mu$ ) - with an intervening air gap is investigated by using the boundary-value technique. The backscattering cross section (BSCS) or the monostatic radar cross section (RCS) has been obtained for both the TM and the TE incident wave polarizations. Analysis based on a mathematical model, namely the Drude-Lorentz dispersion model, for the MTM coating shows that for a certain range of frequencies, the material may behave as either permeability- (or  $\mu$ -) negative (MNG), permittivity- (or epsilon-) negative (ENG), both- (or double-) negative (DNG) or finally as both- (or double-) positive (DPS). The dispersive and lossy characteristics of these materials combined with the added features of an air gap (which can be practically realized by a layer of Styrofoam) seem to indicate that it is possible to achieve an extremely low radar echo width over a broad range of frequencies, particularly for the DNG type MTM. Further investigations on the total scattering cross section (TSCS) for the DNG type MTM, appear to demonstrate that near perfect broadband cloaking is possible with this geometry.

**INDEX TERMS** Cloaking, Conductor, Metamaterials, Radar cross section, Scattering.

## I. INTRODUCTION

Electromagnetic wave scattering characteristics of dielectric coated (single-layered or multi-layered) conductors is a thoroughly researched and analyzed problem [1-5]. Plonus [1] analyzed the far-field backscattering from a perfectly conducting cylinder surrounded by an arbitrary dielectric shell with an intervening air gap. An analysis of far-field backscattering was presented by Rao and Hamid [2] from a multi-layered dielectric coated perfectly conducting cylinder and optimized radial profiles for minimum and maximum backscattering were obtained. Wang [3] analyzed the scattering from a dielectric coated conducting cylinder by obtaining the normalized backscattering width by a high frequency ray optics solution and related the resonances present in the scattering pattern to surface wave resonance phenomenon. Tang [4] presented both theoretical and experimental results of the backscattering cross section from a dielectric coated conducting cylinder. Kodis [5] presented a truncated series solution for the scattered field from a dielectric coated conducting cylinder at high frequencies. An analytical model based on the equivalent moments of a cylindrical PEC rod covered by a conventional dielectric is used to understand and analyze the minimal scattering response of the structure when it is illuminated by a normally

incident plane wave (with the electric field parallel to the cylinder's axis) by Valagiannopoulos et al. [6]. These studies clearly indicate the effects of the thickness and the relative dielectric constant of the coating on the scattered field pattern and on the backscattering cross section.

Metamaterials (MTMs) are a new class of materials that are characterized by negative permittivity ( $\epsilon$ ) and/or negative permeability ( $\mu$ ) values and pioneering work on them was originally reported by Veselago [7] in 1968. The present century has seen a renewed interest in these materials and their possible application in the study of scattering from cylindrical objects [8-18]. References [8-10] lays the foundation for the various kinds of MTM media studied in this work. Smith et al. [11] demonstrated practical realizations and synthesis of these metamaterial-coated conducting cylinders using transformation optics in two dimensions. Although Smith's metamaterial coating design consists of ten Split Ring Resonator (SRR) printed layers with intervening air gaps, the air gaps are functionally intertwined with the SRR layers to give a relative permittivity tensor and a relative permeability tensor, both of which have components that are dependent on radial coordinates, and so the entire structure acts as a single

layered cloak. Li and Shen [12] studied the problem of two-dimensional scattering from a conducting circular cylinder that is coated with a single layer of such a material. They also reported work on the two-layer case, where the first layer surrounding the conductor is a conventional dielectric layer and the second layer is a metamaterial layer. Dispersion was not considered in their studies. Ahmed and Naqvi [13] considered a Perfect Electromagnetic Conductor (PEMC) coated with a layer of non-dispersive metamaterial, presented results of monostatic and bistatic echo widths and compared to those of a dielectric coated PEMC cylinder. Wang and Zhang [14] investigated multi-frequency transparency of a PEC or dielectric cylinder with two layers of dispersive plasmonic coating. Cloaking (or reducing the radar cross section) and maximizing of scattering of a conducting cylinder coated with a single layer of non-dispersive metamaterial was studied by Erturk and Irci [15]. In their study, conditions, and ranges of values of the dielectric constant for achieving transparency and maximizing scattering were reported. Hady and Kishk [16] studied the effects of coating a conducting cylinder with a layer of dispersive metamaterial loaded with conducting strips and presented the results of the effects of changing the arrangement of the strips on the reduction in RCS. Oraizi and Abdolali [17] considered layers of metamaterial and dielectric with dispersive permittivity and permeability properties for both and presented results on scattering cross section on three kinds of media: media consisting of rods only, media consisting of rings only and media consisting of both rings and rods [8-10].

The two-dimensional scattering characteristics of an infinitely long cylindrical conductor coated by a layer of dispersive metamaterial with an intervening air gap and its potential for cloaking applications is investigated in the present article. The scattering characteristics of a coating with no dispersion and an intervening dielectric gap were studied first and the observed results of normalized back scattering were validated by those presented by Li and Shen [12]. The same analysis was carried out by replacing the dielectric layer with an air gap. For cloaking applications, the RCS must be minimized as much as possible over a broad range of frequencies and will only be achieved if scattered power is minimum. It can be conjectured that the lossy nature of the metamaterial layer in view of both  $\epsilon$  and  $\mu$  being complex and the presence of air gap, the scattered power and RCS can be reduced significantly. It is possible in principle to achieve broadband cloaking over a broad range of frequencies, and this may lead to design of sensors for biomedical applications [18].

## II. THEORY

An infinitely long conducting cylinder of radius  $a$ , is covered by a layer of metamaterial in the region  $b \leq \rho \leq c$  (layer-II) and an air gap exists between the two in the region  $a \leq \rho \leq b$

(layer-I), as shown in Fig.1. The structure is illuminated by an EM wave incident at an angle of  $\Phi_0$  with respect to the  $x$ -axis. The incident wave illuminating the structure has either an electric field component  $E_z$  (Transverse Electric) or a magnetic field component  $H_z$  (Transverse Magnetic) parallel to the axis of the cylinder and both polarizations are considered simultaneously. In both cases, the scalar potential function  $A$ , is solved for from the scalar wave equation using the classic separation of variables technique and from it, the rest of the field components within the cylindrical regions and outside are determined. A time-dependence of the form  $e^{-j\omega t}$ , where  $\omega$  is the angular frequency of the incident wave is assumed and suppressed throughout.

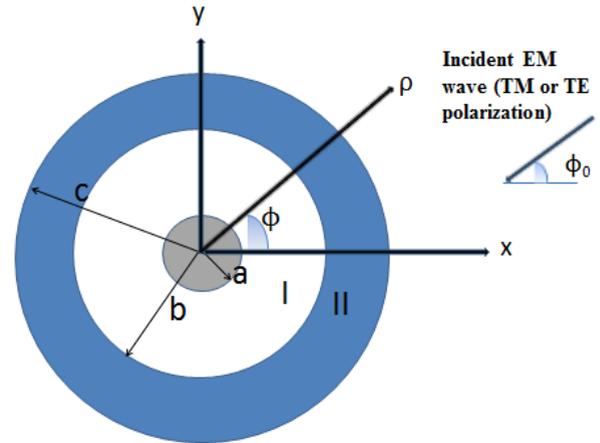


FIGURE 1. Geometry of the problem.

For an incident wave on the cylinder and at an angle  $\Phi_0$  with respect to the positive  $x$ -axis in the  $\rho$ - $\phi$  plane, the incident fields can be written, which are of the form:

$$E_{z, TM}^i = \sum_{n=-\infty}^{\infty} j^n J_n(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (1)$$

$$H_{z, TE}^i = \frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} j^n J_n(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (2)$$

$$E_{\phi, TE}^i = \sum_{n=-\infty}^{\infty} j^n J_n'(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (3)$$

$$H_{\phi, TM}^i = \frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} j^n J_n'(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (4)$$

In the above set of expressions and the scattered field expressions presented below,  $\eta_0$  is the intrinsic impedance of air given by  $\sqrt{\mu_0/\epsilon_0} = 120\pi$  ohms and  $k_0$  is the wavenumber in free space given by  $\omega\sqrt{\mu_0\epsilon_0}$ .  $J_n$  is the  $n$ th order Bessel function of the first kind and the prime notation on the Bessel function indicates the derivative of this function with respect to the entire argument. The scattered fields may be expressed as:

$$E_{z, TM}^s = \sum_{n=-\infty}^{\infty} B_{nTM} j^n H_n^{(1)}(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (5)$$

$$H_{z, TE}^s = \frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} B_{nTE} j^n H_n^{(1)}(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (6)$$

$$E_{\phi, TE}^s = \sum_{n=-\infty}^{\infty} B_{nTE} j^n H_n^{(1)'}(k_o \rho) e^{jn(\Phi - \Phi_0)} \quad (7)$$

$$H_{\Phi, TM}^S = \frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} B_{nTM} j^n H_n^{(1)}(k_0 \rho) e^{jn(\Phi - \Phi_0)} \quad (8)$$

where,  $B_{nTM}$  and  $B_{nTE}$  are the unknown scattering coefficient and the choice of the Hankel function of the first kind satisfies the radiation condition. Next the fields within the layers coating the conducting cylinder will be considered. The fields within the air gap region (layer-I) are given by:

$$E_{Z, TM}^I = \sum_{n=-\infty}^{\infty} [C_{nTM}^I J_n(k_1 \rho) + D_{nTM}^I Y_n(k_1 \rho)] e^{jn\Phi} \quad (9)$$

$$H_{Z, TE}^I = \frac{j}{\eta_1} \sum_{n=-\infty}^{\infty} [C_{nTE}^I J_n(k_1 \rho) + D_{nTE}^I Y_n(k_1 \rho)] e^{jn\Phi} \quad (10)$$

$$E_{\Phi, TE}^I = \sum_{n=-\infty}^{\infty} [C_{nTE}^I J_n'(k_1 \rho) + D_{nTE}^I Y_n'(k_1 \rho)] e^{jn\Phi} \quad (11)$$

$$H_{\Phi, TM}^I = \frac{j}{\eta_1} \sum_{n=-\infty}^{\infty} [C_{nTM}^I J_n'(k_1 \rho) + D_{nTM}^I Y_n'(k_1 \rho)] e^{jn\Phi} \quad (12)$$

where  $J_n$  and  $Y_n$  are Bessel functions of first and second kinds,  $\eta_1$  is the intrinsic impedance of layer-I given by  $\sqrt{\mu_{r1} \mu_0 / \epsilon_{r1} \epsilon_0}$  and  $k_1$  is the wavenumber in layer-I given by  $\omega \sqrt{\mu_{r1} \mu_0 \epsilon_{r1} \epsilon_0}$ .  $\mu_{r1}$  and  $\epsilon_{r1}$  are both equal to 1 in layer-I, as layer-I is an air gap, and so  $\eta_1 = 120\pi$ . The fields within the layer-II are written similarly as:

$$E_{Z, TM}^{II} = \sum_{n=-\infty}^{\infty} [C_{nTM}^{II} J_n(k_2 \rho) + D_{nTM}^{II} Y_n(k_2 \rho)] e^{jn\Phi} \quad (13)$$

$$H_{Z, TE}^{II} = \frac{j}{\eta_2} \sum_{n=-\infty}^{\infty} [C_{nTE}^{II} J_n(k_2 \rho) + D_{nTE}^{II} Y_n(k_2 \rho)] e^{jn\Phi} \quad (14)$$

$$E_{\Phi, TE}^{II} = \sum_{n=-\infty}^{\infty} [C_{nTE}^{II} J_n'(k_2 \rho) + D_{nTE}^{II} Y_n'(k_2 \rho)] e^{jn\Phi} \quad (15)$$

$$H_{\Phi, TM}^{II} = \frac{j}{\eta_2} \sum_{n=-\infty}^{\infty} [C_{nTM}^{II} J_n'(k_2 \rho) + D_{nTM}^{II} Y_n'(k_2 \rho)] e^{jn\Phi} \quad (16)$$

If layer-II is made of an arbitrary dielectric,  $\eta_2$  (intrinsic impedance of layer-II) is given by  $\sqrt{\mu_{r2} \mu_0 / \epsilon_{r2} \epsilon_0}$  and the wavenumber in layer-II,  $k_2$ , is  $\omega \sqrt{\mu_{r2} \mu_0 \epsilon_{r2} \epsilon_0}$ . When layer-II is a metamaterial, these relations will be modified accordingly. The unknown coefficients in Equation 5-16 are determined by using boundary conditions at the three boundaries ( $\rho = a, \rho = b$  and  $\rho = c$ ). By substituting the field expressions and applying the large argument approximation of the Hankel function, the normalized *total scattering cross section (TSCS)* can be obtained for the two polarizations and are given by:

$$\frac{\sigma_{TM}}{\lambda_0} = \frac{2}{\pi} \left| \sum_{n=-\infty}^{\infty} B_{nTM} e^{jn(\Phi - \Phi_0)} \right|^2 \quad (17)$$

$$\frac{\sigma_{TE}}{\lambda_0} = \frac{2}{\pi} \left| \sum_{n=-\infty}^{\infty} B_{nTE} e^{jn(\Phi - \Phi_0)} \right|^2 \quad (18)$$

For calculating the TSCS,  $\Phi$  is the full range of scattering observation angles ( $0^\circ \leq \Phi \leq 360^\circ$ ) and  $\Phi_0$  is the incident angle ( $\Phi_0 = 0$ ). A particular case of interest is when the two angles  $\Phi$  and  $\Phi_0$  coincide, thus giving us the BSCS:

$$\frac{\sigma_{TM}}{\lambda_0} = \frac{2}{\pi} \left| \sum_{n=-\infty}^{\infty} B_{nTM} (-1)^n \right|^2 \quad (19)$$

$$\frac{\sigma_{TE}}{\lambda_0} = \frac{2}{\pi} \left| \sum_{n=-\infty}^{\infty} B_{nTE} (-1)^n \right|^2 \quad (20)$$

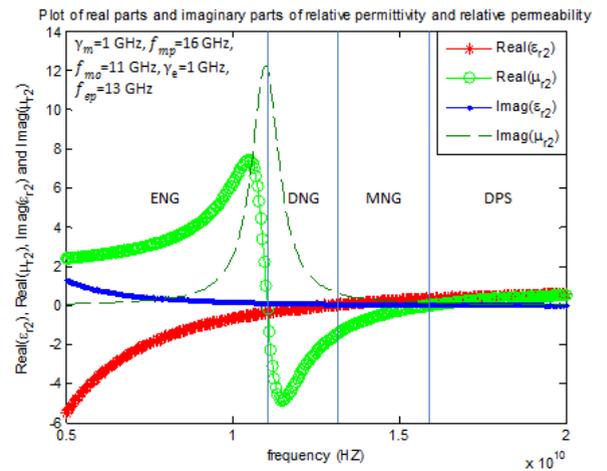
In the present investigation, three types of dispersion models for MTMs are considered. These models have the relative

permittivity and relative permeability expressed as functions of frequency, electric and magnetic collision frequencies  $\gamma_e$  and  $\gamma_m$ , electric and magnetic plasma frequencies  $f_{ep}$  and  $f_{mp}$  and the low frequency edges of the electric and magnetic forbidden bands  $f_{e0}$  and  $f_{m0}$  [10]. Three different types are considered - rings only, rods only and both rings and rods - with their corresponding dispersive constitutive parameters are given below [17]:

**TABLE I.** List of three types of Metamaterials used for the simulations. Frequencies are in GHz [17]

Class of materials	Permittivity Model	Permeability Model	Parameter ranges
Rods only	$\epsilon_{r2} = 1 - \frac{f_{ep}^2}{f^2 + j\gamma_e}$	$\mu_{r2} = 1$	$1 \leq f_{ep} \leq 30$ $0.001 \leq \gamma_e \leq 5$
Rings only	$\epsilon_{r2} = 1$	$\mu_{r2} = 1 - \frac{f_{mp}^2 - f_{m0}^2}{f - f_{m0} + j\gamma_m}$	$1 \leq f_{m0} \leq 30$ $f_{mp} \sim f_{m0} \pm [0.1, 5]$ $0.001 \leq \gamma_m \leq 5$
Rods and Rings	$\epsilon_{r2} = 1 - \frac{f_{ep}^2}{f^2 + j\gamma_e}$	$\mu_{r2} = 1 - \frac{f_{mp}^2 - f_{m0}^2}{f - f_{m0} + j\gamma_m}$	Same as above

The relations in Table-1 model the constitutive parameters of the metamaterial layer and are based on the Drude and Lorentz model. Shown below in Fig. 2 is a plot of the real and imaginary parts of the constitutive parameters for the rods and rings type media in the frequency range of 5 – 20 GHz.



**FIGURE 2.** Real parts of relative permittivity,  $\epsilon_{r2}$ , and permeability,  $\mu_{r2}$ , of Rods and Rings media vs. frequency.

An examination of Fig. 2 shows that for certain frequency bands, the metamaterial layer may act as DPS ( $\text{Re}(\epsilon) > 0$ ,  $\text{Re}(\mu) > 0$ ), DNG ( $\text{Re}(\epsilon) < 0$ ,  $\text{Re}(\mu) < 0$ ), ENG ( $\text{Re}(\epsilon) < 0$ ,  $\text{Re}(\mu) > 0$ ) or MNG ( $\text{Re}(\epsilon) > 0$ ,  $\text{Re}(\mu) < 0$ ). For the time dependence chosen in this paper and to maintain consistency with Poynting's theorem, the imaginary parts of the two parameters need to be positive. The characteristic parameters of the coating layer are therefore given by [17]:

$$\varepsilon = \pm\varepsilon' + j\varepsilon'' \quad (21)$$

$$\mu = \pm\mu' + j\mu'' \quad (22)$$

$$k = \pm k' + jk'' \quad (23)$$

$$\eta = \eta' \pm j\eta'' \quad (24)$$

It should be noted that the lossless DNG case  $k = -k'$  and for the lossless ENG case  $\eta = -j\eta''$ .

### III. RESULTS

In this section, some representative numerical results on the backscattering cross section of the proposed structure for both TM and TE polarizations of the incident wave are presented. If the BSCS,  $\sigma$ , can be made significantly below the incident wavelength (swept), over a range of frequencies, then the conducting object could be considered successfully cloaked. The present study includes the dispersive effects of the MTM coating to see whether this leads to a broad range of frequency where  $\sigma$  remains consistently below a fraction of a wavelength. A scaled BSCS value of -10 dB is the threshold value and if it remains below that value for a sizeable bandwidth, the conducting cylinder can be considered to have been cloaked over a broad band. The signs of the real parts of the constitutive parameters vary with different frequency bands. This leads to interesting phenomenon when a coated PEC cylinder is illuminated, and a frequency sweep is made. BSCS minimization and maximization occurs at different frequency bands. The focus is on BSCS minimization over a broad range of frequencies which will be shown by some of the results presented below. The values of  $\gamma_c$ ,  $\gamma_m$ ,  $f_{ep}$ ,  $f_{mp}$ ,  $f_{e0}$  and  $f_{m0}$  are same for all three cases. In all the simulations a PEC cylinder of radius 50mm is being cloaked and a frequency spectrum of 0.1 to 9 GHz is considered. The SW is given in dB unit.

#### A. RODS ONLY

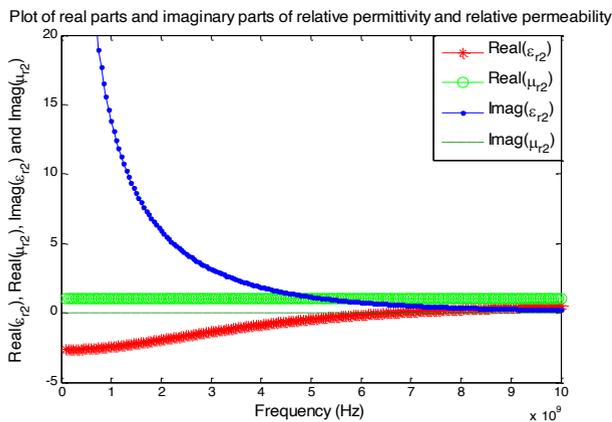


FIGURE 3. Real and imaginary parts of relative permittivity,  $\varepsilon_{r2}$ , and permeability,  $\mu_{r2}$ , of Rods only media versus frequency for  $\gamma_e = 4.05248$  GHz,  $f_{ep} = 7.71387$  GHz.

Figure 3 shows the variation of the real and imaginary parts of  $\varepsilon$  and  $\mu$  for the rods only medium. It is observed that for the frequency range of approximately 0.1 GHz to 6.5 GHz the layer acts as ENG type and for the rest of the spectrum as DPS type, for the set of material parameters defined. From Fig. 4, it can be observed that as the thickness of the air gap is increased, the BSCS changes non-monotonically for the TM polarization case. Until roughly 5.75 GHz, the backscattering characteristics for  $b/a=1.1$  and  $1.25$  are almost at the same levels and for  $b/a=1.2$ , the backscattering level hovers close to the -5 dB mark. There is no cloaking at all for the TM case, with ENG type of material coating under these conditions as there is no backscattering on or below -10 dB for any part of the frequency range above 0.5 GHz.

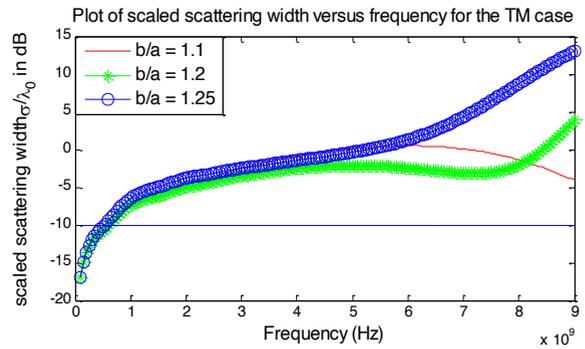


FIGURE 4. Normalized backscattering width (TM polarization) versus frequency of Rods only media coating for  $b/a = 1.1, 1.2$  and  $1.25$  with fixed  $c/a = 1.3$

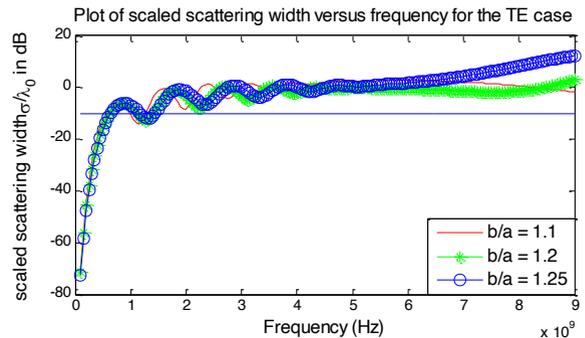
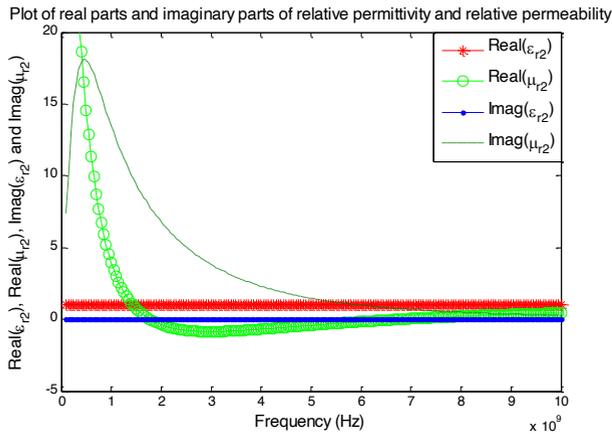


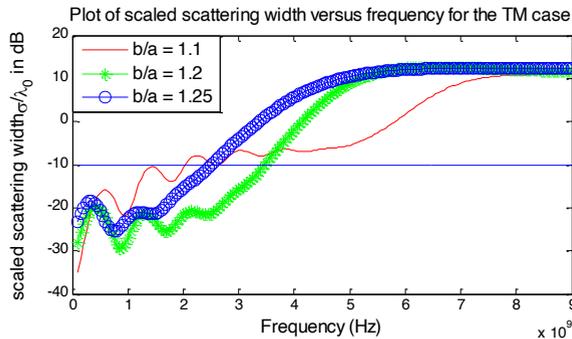
FIGURE 5. Normalized backscattering width (TE polarization) versus frequency of Rods only media coating for  $b/a = 1.1, 1.2$  and  $1.25$  with fixed  $c/a = 1.3$

In the TE case (Fig. 5), it can be observed that the BSCS for all three air gap sizes are hovering around 0 dB roughly in between 1.5 to 6.5 GHz. The observed oscillatory behavior suggests the presence of internal resonances and the backscattering characteristics are consistent with previously obtained results of Wang [3] and Kodis [5] for backscattering from a dielectric-coated conducting cylinder. Cloaking is not achievable under these conditions as no part of the backscattering characteristics is below -10 dB beyond 1.5 GHz.

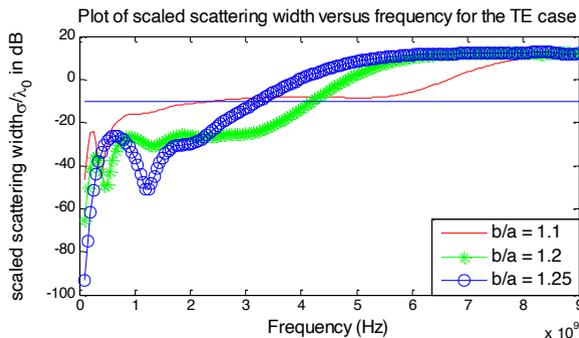
**B. RINGS ONLY**



**FIGURE 6.** Real and imaginary parts of relative permittivity,  $\epsilon_{r2}$ , and permeability,  $\mu_{r2}$ , of Rings only media versus frequency for  $\gamma_m = 4.78006$  GHz,  $f_{mp} = 8.34629$  GHz,  $f_{mo} = 1.4311$  GHz



**FIGURE 7.** Normalized backscattering width (TM polarization) versus frequency of Rings only media coating for  $b/a = 1.1, 1.2$  and  $1.25$  with fixed  $c/a = 1.3$

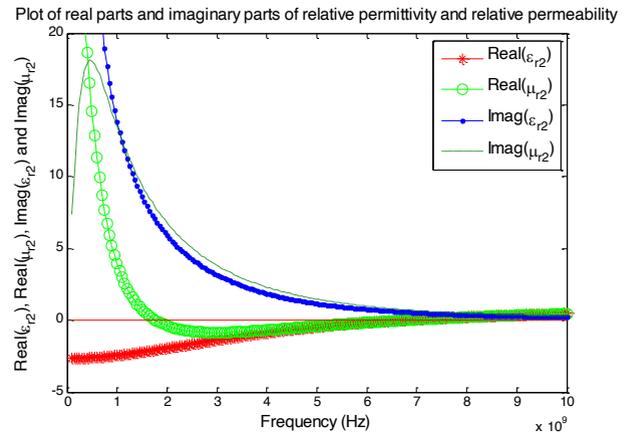


**FIGURE 8.** Normalized backscattering width (TE polarization) versus frequency of Rings only media coating for  $b/a = 1.1, 1.2$  and  $1.25$  with fixed  $c/a = 1.3$

In the case of rings only type medium, the constitutive parameters of the layer assumed are shown in Fig. 6. The layer of MTM behaves as an MNG type medium for the frequency range of 1.75 GHz to 6.5 GHz, and as DPS type for the rest of the spectrum. For the TM incidence case for the rings only medium coating (Fig. 7) it can be seen the BSCS is reducing non-monotonically as the air gap thickness is increasing. The backscattering levels for all three air gap

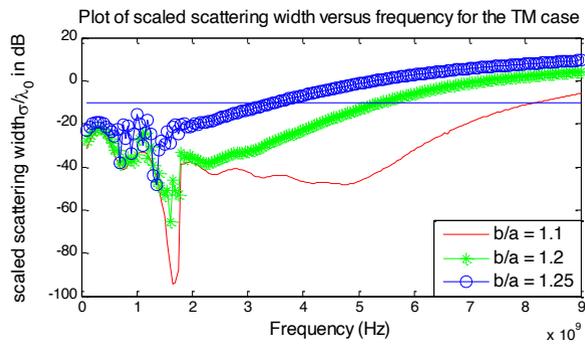
sizes are below -10dB until roughly about 2 GHz, with  $b/a = 1.2$  having backscattering below -10 dB until 3.5 GHz, suggesting that cloaking can be achieved with this configuration for a sizeable bandwidth. This contrasts with the TM case for rods only or ENG medium. Also, a suppression of resonance phenomenon occurs as the air gap thickness is increased. Furthermore, beyond roughly 3.5 GHz, there is a monotonic increase in RCS noticeable as air gap size increases. In the case of the other polarization (Fig. 8), it can be observed the backscattering levels for all three air gap sizes are below -10dB until roughly about 2.25 GHz, with  $b/a = 1.2$  having backscattering below -10 dB until 4 GHz, suggesting that cloaking can be achieved with this configuration as well for a sizeable bandwidth. Again, beyond roughly 4.3 GHz, there is a monotonic increase in RCS noticeable as air gap size increases. For both polarizations, the backscattering levels seem to reach a threshold value as frequency increases to 9 GHz.

**C. RINGS AND RODS**

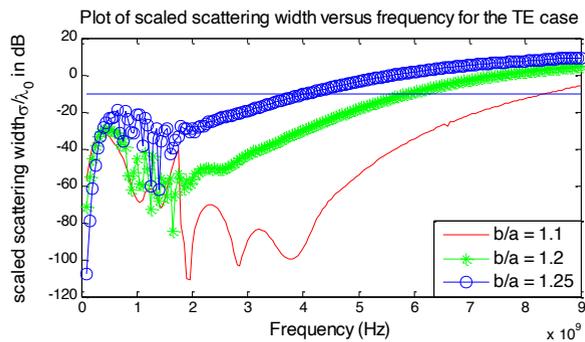


**FIGURE 9.** Real and imaginary parts of relative permittivity,  $\epsilon_{r2}$ , and relative permeability,  $\mu_{r2}$ , of Rings and rods media versus frequency for  $\gamma_m = 4.78006$  GHz,  $f_{mp} = 8.34629$  GHz,  $f_{mo} = 1.4311$  GHz,  $\gamma_e = 4.05248$  GHz,  $f_{ep} = 7.71387$  GHz

Figure 9 shows the variation of the real and imaginary parts of  $\epsilon$  and  $\mu$ . At frequencies below 1.75 GHz, the medium behaves more like ENG medium ( $\epsilon$  is negative, but  $\mu$  is not) and above the frequency of 6.5 GHz, both  $\epsilon$  and  $\mu$  are small and approaching zero. In between these two extreme cases, the medium behaves as a DNG material and the air gap size is expected to help the broadband cloaking. For the TM polarization (Fig. 10) we observe that the BSCS remains much lower than -10 dB ( $0.1\lambda_0$ ) with increasing bandwidth as air gap size reduces. The highest bandwidth over which cloaking is achieved is with  $b/a=1.1$ , which ranges from 0.1 GHz to roughly 8 GHz. The overall pattern is monotonically increasing with air gap size. An exact similar pattern is observed with the TE case (Fig.11) and thus rings and rods media seem to be the choice of the kind of MTM for cloaking purposes for both polarizations.

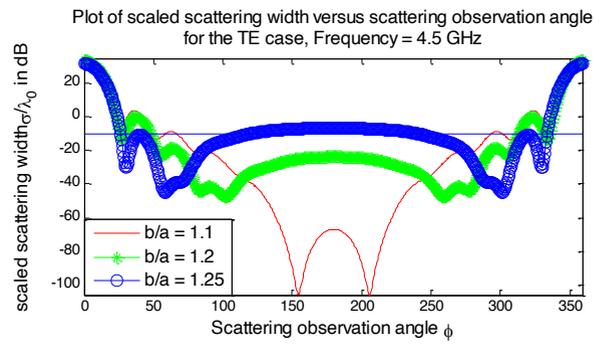


**FIGURE 10.** Normalized backscattering width (TM polarization) versus frequency of Rings and rods media coating with  $c/a = 1.3$

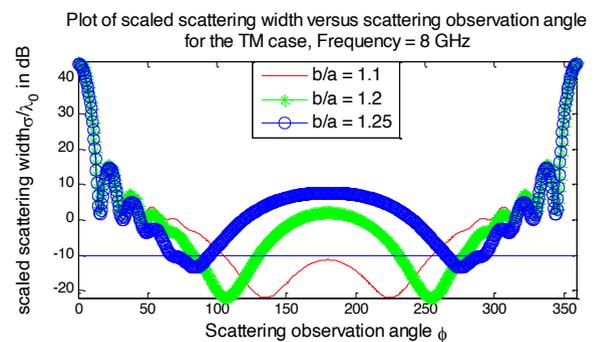


**FIGURE 11.** Normalized backscattering width (TE polarization) versus frequency of Rings and rods media coating with  $c/a = 1.3$

A further dissection is presented with the total scattering cross section (TSCS) patterns for both polarizations at different frequencies (Fig. 12 and 13). Figure 12 shows the TSCS pattern for the TE case at a frequency of 4.5 GHz. The values of the scattering levels for all three air gaps at  $\Phi = 180^\circ$  correspond to the values of the backscattering levels at 4.5 GHz of Fig. 11. It is observed from Fig. 12 that for  $b/a = 1.1$  and  $1.2$  the scattering levels are much below the  $-10$  dB mark for a wide range of angle. In fact, it is much below  $-10$  dB for most of the patterns except for the main lobe and the first side lobe and a much smaller portion of the second side lobe, which is a strong indication for effective cloaking. For a higher frequency of 8 GHz with the other polarization (Fig. 13), it can be observed that for  $b/a = 1.1$ , cloaking can still be achieved for a significantly wide range of angles ( $100^\circ \leq \Phi \leq 250^\circ$ ), which include the backscattering direction. These are strong indication that rings and rods provide the best performance for cloaking under these conditions.



**FIGURE 12.** Total Scattering Cross Section for TE incidence at  $f_0 = 4.5$  GHz with rings and rods media coating.



**FIGURE 13.** Total Scattering Cross Section for TM incidence at  $f_0 = 8$  GHz with rings and rods media coating.

#### IV. CONCLUSION

In this paper we have presented results of the scattering characteristics of a perfectly conducting cylinder coated with three different kinds of media (media with rings only, media with rods only and media with both rings and rods) which behave as MTMs and having an air gap between the conductor and the coating. Two different incident polarizations are considered (TM and TE). The work considers a deep look at the case where the intervening layer is an air gap (practically realized by Styrofoam) and the effects of the size of this air gap on the reduction of the RCS. Furthermore, examples are provided of the contrasting behavior of three different kinds of MTM used for the outer layer (having the same set of characterization parameters used by Drude and/or Lorentz models for all three kinds).

We see anomalous behavior of the backscattering for all three types of MTM coatings and when the air gap size is changed. The broadest bandwidth over which cloaking may be achieved is shown by the media which consisted of both rings and rods for both polarizations, as it approximates the DNG MTM. Analyzing this case further, it was shown by calculating the total scattering cross section that for fixed frequencies cloaking may be achieved for a broad range of scattering angles. Furthermore, effects of the size of air gap on the range of scattering angles for which cloaking is achieved were

studied. All these results point to interesting possibilities of achieving near perfect cloaking of a cylindrical conductor with MTM coatings and having an intervening air gap.

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