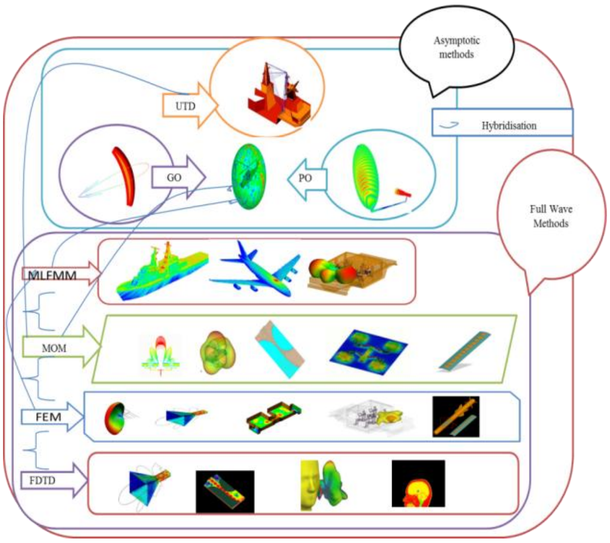
is a challenge to conventional MOM approach. MLFMA can be implemented in parallel processing using iterative Eigen



Source of the Figure: 3[21]

Figure : 3. Applications of computational Electromagnetics

solvers, yield desired performance in CM analysis. The applications of various computational electromagnetic techniques are depicted in Figure: 3.

The commercial codes like HFSS®, CST®, FEKO®, XFDTD®, Empire®, SEMCAD® uses differential methods like finite difference method and finite element method. FEKO, WIPL-D® (IE3D, Sonnet, Designer etc. – 2D) are using integral methods like method of moments.

The Survey of CEM method is compared in Table 1: Survey of these methods based on issues associated with each method like MOM, FEM, FDTD and Hybridization of Techniques is discussed. Correspondingly solutions to these issues are also addressed in this Table.

3. Meshless and Meshfree methods

Mesh less method are used in arriving at numerical solution for problems where meshes are not necessary however they use mesh only once in final stage to solve linear equations. In order to remove meshes, we require designing of fitting policies for scattered data in multi-dimensional spaces that results in definition of mesh less shape functions [53]. Point interpolation method, radial point interpolation method and Shepard approximation are few natural neighboring methods used to define shape function. The Shepard approximation help in achieving good accuracy and considerable computing time [54]. These problems are solved by expanding unknown field variable over such shape function and also reduce number of unknowns. This method has few drawbacks in terms of accuracy and computational time. Sharp discontinuity and difficult simulation are dealt in mesh less method. The various advantages of mesh less method are increased support domain, change in basis functions, adopting nodal densities. The meshless methods like Meshless Local Petrov- Galerkin (MLPG) , Local Boundary Integral Equation (LBIE) and Meshless Local Boundary Equation (MLBE) are used in the analysis of microstrip antennas [53].

Table 1: Survey on CEM methods

Categories	Issues	Solution
MOM Based Techniques	Singularity, Low frequency break down, Charge cancellation	Singularity extraction, MOM elliptic formulation, Preconditioners, Charge recovery
FEM based Techniques	Internal interface issue, Spurious solution , Difficult in solving electrically large problem	HLU, DDA-ABC preconditioners. DDA- parallelization.
FDTD based Techniques	Truncation error at each step , synchronization issues, interpolation error	CFL limit- step size small, Semi implicit schemes- step size large, RK-HO-FDTD.
Hybridization of Techniques	Ill conditioning effects Multiscale problems	DDA-ABC preconditioner MLFMA with some FFT-based algorithm

All these methods decrease computational time by changing domain integrals to boundary integrals. Amongst these three methods it is expected that MLBE is fastest. The comparison of all these methods for thin microstrip with coax fed and line fed shows same convergence rate. LBIE has least condition number and MLBE needs low CPU time.

Mesh free method usually bypasses mesh generation. Mesh free methods are used in solving computationally difficult problems at the cost of computational time and programming effort. The mesh free method has several benefits over FEM and FDM such as Overlapping domain gives much support and gives good approximation.

4. GPU acceleration

In spite of development in GPGPU computing and speeding of parts of programs or using easy problems many factors that make problem robust are considered before implementing this technique to commercial software. The expected solution needs increased computational power and therefore time required for solution is reduced. Successful GPU implementation needs code implementation and optimization depending on problem time regardless of whether it is my memory bound or compound bound. GPGPU has both hardware capability like c/c++ and software programming. This method is used in FEKO and choice of solution depends on electrical size of problem and intricacy of materials that are simulated. The Challenges in GPU acceleration are versatility, reliability and reproducibility, variety of CEM methods and software and design decisions. Multi GPU and heterogeneous system increase computational performance are discussed in [55].

Challenges faced by GPU acceleration are discussed in [56].

1. **Flexibility, consistency and reproducibility**

   Commercial software settings demands accurate results without exceeding allocated memory for GPU acceleration. They require additional computational resources when they run out of GPU memory, in such cases the switching algorithms transfer control to CPU.

2. **Wide variety of computational electromagnetics methods**

   GPU faces several challenges in parallelization techniques like MPI, Open MP and GPU computation. GPU acceleration has similar approaches for MOM and FEM, In spite of their differences (Linear system is dense matrix for MOM and sparse for FEM). However realized speed will differ for dense and sparse GPU computation.

3. **Software used and design decisions**

   GPU acceleration on commercial CEM software are based on decisions such as language for software implementation and low-level program flow such that it maps well to GPU architecture

Open CL, Open MP and CUDA are few programming languages preferred and partially used in commercial soft wares like FEKO. GPU programming is C/C++ based and acceleration of this is FORTRAN based. Any modification of codes may result in bugs; further introduce need for testing, tuning and software verification

GPU processing for MOM matrix experiences run time that is quadratic and solution that is cubic. In case of larger problem matrix solution will dominate the run time. Speed up for MOM code needs resources to be invested due to complex nature of code.

In case of FEM GPU acceleration must be overlooked. The total simulation time of FEM include construction of preconditioner and solution to sparse matrix. Use of right preconditioner decreases the solution time in comparison to direct sparse solvers. Single GPU is used for small problems and GPU cluster is used for large problem. Iterative solvers show performance improvement in cases where GPU memory is a limitation and they provide a speed up of 50%.

FDTD based GPU acceleration is no hybridization is involved so computational resources are reduced. Optimization of GPU based FDTD is done by exploiting shared memory, achieves global memory coalesced accesses, employing texture caches, use of build in arrays, properly arrange computation in third domain. GPU provides speed up of ten times compared to CPU. Parallelization of 75% is achieved by overlapping computation and communication. Ray Launching-Geometric optics used UTD, PO, RL-GO collectively resulting in Shooting and Bouncing Ray (SBR) for dealing with large objects and is well suited for GPU acceleration. CUDA code is run through GPU compiler to obtain accelerated GPU implementation. Stack size must be addressed well to deal with GPU limitation in terms of difficulty and recursive code.

Further acceleration of these methods is done by using numerical algorithms through hybridization for all methods except FDTD.

5. Hybridization

MOM faces difficulty in modeling inhomogeneous, interior of conducting enclosures and dielectrics with non-linearity. FEM is not suited for efficient modeling of thin wires, large radiation problems and Eigen value problem due to its unstructured mesh. FDTD is difficult in modeling structure with sharp edges. GTM and UTD techniques are not suitable for problems that need accurate measurement of surface currents. It is obvious from the above discussion that none of the numerical techniques can solve all EM problems. All these techniques fail to cater the needs of printed radiation models that have all these structures. The most appropriate solution found by the researchers is to club two or three techniques and produce one code i.e. Hybridization code. Hybridization techniques involve combing two or more techniques into a single code. Various hybridization techniques are discussed in the forthcoming section.

5.1. Hybrid MOM - FDTD

FDTD method accomplishes propagation simulations excellently but not suited for modeling complex metallic structures like antennas. On the other hand the Method of Moments (MOM) is ideal for modeling complex metallic structures and is not very well suited for penetrating into such structures, hybrid MOM /FDTD method is used in application that require penetration into these structures eg. human tissue [57]. Antenna and scattering problems could be resolved using hybrid MOM and FDTD based on IMR (Iterative Multi-Region) technique [58]. IMR divides the domain problem into separate sub regions. In a problem with thin wire and scatterer MOM is used to solve thin wire antenna while the other region can be solved using FDTD solutions. Iterative algorithm helps in achieving the solution for combined sub regions. Radiated fields arising from MOM region due to current distribution on the antenna helps in interaction of two sub regions. Since the FDTD is a time domain solver, fields emanating from the MOM region that excite the FDTD region needs to be changed into time domain waveforms. This method helps in achieving reduction in the memory storage requirements and computation time.

Hybrid MOM-FDTD method employing the Asymptotic Waveform Evaluation (AWE) technique [59] involves swapping back and forth information between the MOM (DFT) and FDTD (IDFT). The AWE technique in the MOM domain is implemented to reduce the computational time needed for wide-band analysis. Frequency hopping technique is suggested for choice of expansion points in AWE technique. The computational time is reduced from 3 hours 18 min to 1 hour 4 min by using AWE technique.

5.2. Hybrid FDTD - PO

Radiating planar antennas in the existence of large conductive structures are analyzed using Hybrid FDTD and PO [60]. Surface equivalence theorem is used to combine FDTD and PO and spatial interpolation technique is used to enhance computational efficiency of the proposed approach. The idea of using this technique is to calculate samples of the electric and magnetic fields on a comparatively coarse spatial grid over the Huygens surface and then to use interpolation for obtaining field values on the required fine grid. FDTD regions enclose the antennas and inhomogeneous dielectric objects surrounded by Huygens Surface. Conducting bodies in frequency domain are analyzed using PO. Memory storage and CPU time are saved by using this technique.

5.3. Hybrid FEM - FDTD

Hybrid FEM - FDTD is aids to present modified equivalent surface current [61] by means of equivalent principle theorem to extend the field transformation. FDTD gained popularity due to its simplicity and efficiency. However compromise is made in terms of accuracy. FEM permits good estimates of complicated boundaries and with edge elements it performs well for Maxwell's equations but needs further memory hybrid that applies FDTD in large volumes. FEM is difficult to be applied for problems with large dimensions but FDTD can handle this even for penetrable structures. 70% of the required memory locations of the field points between the two domains are saved along with increase in speed for updating boundary equations inside the FDTD method [62].

5.4. Hybrid MOM - PO

MOM could be hybridized with FEM, FDTD, TLM, AEM, and PO. In case of MOM - PO hybrid MOM method is used in small, resonating structures near edge while PO is used for large and smooth regions. MOM /PO hybrid is preferred for modeling large reflector antenna instead of MLFMM. Reflector is modeled using PO and feed of the reflector is accurately modeled using MOM.

5.5. Hybrid FEM - MOM

In spite of the advantage of FEM like mesh adaption and mesh refinement that improves accuracy, there is disadvantage in this technique in case of mesh termination. Mesh termination issue arises when radiation condition are enforced in open region. Various methodologies have been used for mesh termination but FEM-MOM hybrid is the best method. FEM uses curl and field based Whitney element while MOM uses divergence and current based RWG element.

5.6. Hybrid FDTD with two level atomic systems

Hybrid of atomic systems emerged from coupling rate equations and FDTD. EM fields and atoms population density are updated and leads to the significant reduction of computational effort for the analysis of absorbing materials [63]. Short FDTD simulations allow updating the population density of the system at a much longer time scale than a single cycle of the wave

5.7. Hybrid FDTD with DGF formulation

Hybridization of FDTD method with formulation of DGF (Discrete Green Function) [64, 65] has limited utility due to computation overhead. It anticipates new fast methods for DGF generation. However this hybrid formulation finds its application in antenna and disjoints domains. The antenna is modeled by DGF formulation of FDTD, while scatterer is modeled with FDTD. Implementation of this method in parallel processing using iterative Eigen solvers yield desired performance in CM analysis.

5.8. Advanced Hybridization Schemes

FE-BI-MLFMA which is derived from FE-BI uses absorbing boundary condition to do first approximation by FEM. The solution based on FETI faces issues in terms of numerical scalability and computational difficulty. Although, preconditioners are utilized to reduce computational time enabling fast convergence towards desired solution with less number of iterations, algebraic preconditioners need complex factorization making them unsuitable in this regard. However DDA-ABC preconditioner employing iterative solver using DDM satisfies the need. This method finds its application in antenna arrays and large lossless objects. Computational time can be reduced by incorporating ID algorithm and the solution is obtained by combining direct solvers with iterative solvers. The proposed method with ID exhibits more accurate, efficient and robust result. Further it decreases the peak memory requirement while it maintains the number of the final skeleton directions the same as or less is applicable in 3-D composite objects [66].

Multiscale problems are analyzed using various hybrid techniques like MLFMA, LFFIPWA effective in solving sub wavelength breakdown with high accuracy and low efficiency compromising the speed. MLIPFFT-MLFMA (Multilevel Interpolatory Fast Fourier Transform –MLFMA) is a broadband method with higher efficiency. ID-MLFMA, MLFMA-ACA and MLFMA with some Fast Fourier Transform based method solves multiscale problem in large structures. On the other hand in terms of memory storage and computing resources ID-MLFMA and MLFMA-ACA techniques are capable of treating multiscale problem [67].
MLFMA-PO is a hybrid technique used in scenarios where geometry with fine details needs to be modeled with

accuracy and efficiency. MLFMA is an acceleration of MOM that is responsible for modeling finite parts with accuracy. PO can model large structures with efficiency. Hybrid MLFMA-PO can be used in applications involving large scattering and large radiation problems like certain critical parts of antenna [68].

Parallelization improve the speed and efficiency of a process, also this is implemented in hybrid parallel open MP -VALU (Vector Arithmetic Logic Unit) MLFMA to achieve the same. In contrast to GPU acceleration the above cited technique does not demand extra device due to the presence of essential VALU inside CPU. The hybrid SPMD-SIMD parallel scheme is not supported by hybrid parallel Open MP-VALU MLFMA method [69]. Hybrid MPI and Open MP parallel programming technique is one such technique where in BOR-MOD-CFIE code is subjected to parallelization to enhance performance of MOD method in terms of memory, accuracy and CPU time [70].

MLFMA with PO to form hybrid finds its application in analyzing scattering and radiation problems for electrically large structures. The proposed hybrid uses MLFMA, accelerated version of MOM for analyzing certain structures where complex linear equation are solved by iterative solvers explicitly and PO, on the other hand to enhance the accuracy dealing with scattered fields. The application involving fast RCS prediction of electrically large target are accomplished using hybridization of GO/PO accelerated by open graphics library enabling accuracy in capturing creeping effect and wave diffraction effects [71].

High frequency approximation method called shooting and bouncing ray is used in applications involving PEC and dielectric using CUDA. This method uses physical approximation, current based method in SBR owing to its capability to penetrating into objects for applications like RADAR and ISAR [72]

Juan Chen [73] proposed a hybridization method based on WCS-PSTD (Weakly Conditionally Stable – Pseudo Spectral Time Domain).This method is formulated to deal with electrically large object with continuity between subdomains implies less memory and less computational time. For photonic crystal this method is better than FDTD. However, accuracy of the proposed method that largely depends on step size and is not applicable to surfaces with discontinuity.

Time domain application of Hybridization like DGTD and TDBI incorporates calculation of truncated boundary while truncation is brought about by ABC or EAC, PML. Flux produced by truncated boundary is responsible for communication between the domains and determines shape of scatterer by imposing physical radiation. DGTD truncated by EAC makes radiation condition elastic than all DGTD systems to address EM scattering problems [74].

TABLE 2: Techniques for performance up gradation of CEM methods

Category	Techniques for performance upgradation of CEM			
	MOM®	**FEM®**	**FDTD®**	**Hybridization ®**
Accuracy	Singularity extraction, MOM elliptic formulation	1.H-LU - computation cost and accuracy	CFL limit- step size small Semi implicit schemes- step size large RK-HO-FDTD.	Hybrid techniques accelerated by open graphics library enabling accuracy DDP-FE-BI-MLFMA
Memory	Incremental multilevel filling and sparsification	Nested Dissection- DDM. Semi Implicit Schemes . -	Sub cell algorithm, RK-HO-FDTD.	ID-MLFMA and MLFMA-ACA, Open MP parallel programming
CPU time	Singularity extraction, Paralleli-zation	DDA-parallelization-increases speed	ADI-FDTD, LOD-FDTD – paralleli-zation, Sub cell algorithm, RK-HO-FDTD.	Hybrid MPI and Open MP parallel programming technique, WCS-PSTD, Preconditioner, ID algorithm. AWE technique in MOM-FDTD hybrid reduce resource

The strengths and weakness of different CEM techniques were discussed. In summary, the following were observed: FDTD is a time stepping phenomena used in the analysis of domains like Debye dispersive media. Discretization of this media imposes error due to truncation, thereby increasing dependency of accuracy to Courant, Friedrichs, Lewy (CFL) limit [8, 10].

This dependency is mitigated by using Semi implicit schemes (SIS) [12]. Novel Runge-Kutta algorithm makes FDTD stable, dispersive and convergent [18]. FEM schemes faced problems in modeling 3D objects, Eigen value problem and thin wires. Domain decomposition method (DDM) based FETI-DPEM solves FE system and makes good in terms of convergence and scalability with aid of direct FEM solvers. Nested Dissection assures less memory and CPU time, while H-LU provides accuracy and fewer resources when used along with FEM. Table 2 depicts techniques for performance upgradation of CEM methods.

Iterative solvers face difficulty in convergence due to existence of PML. However DtN approximation, preconditioners promotes convergence [25]. DDM makes it applicable to large domain problems. MLFMA acceleration incorporates accuracy as depicted DDP-FE-BI-MLFMA technique, is suitable for all domains [29]. MOM needs the domain to be well conditioned and error controllable. Ill conditioning effects could be reduced by potentially splitting the domain or by the use of preconditioners. Error could be controlled by using semi implicit schemes and formulation like MOM elliptic approach [36]. Various hybrid combinations and advanced hybridization like FE-BI-MLFMA with limited scalability and computational complications [28,29], ID-MLFMA [67], MLFMA-ACA that treat multiscale problems [67], Open MP VALU[69], MLFMA, WCS-PSTD[73], DGTD–TDBI with truncated by EAC shows better performance in terms of accuracy, convergence and CPU time [74]. Most widely used softwares are CST®[75] based on Finite Integration Technique, HFSS® [76] based on FEM and FEKO ® [77] based on MOM and FEM.

6. Future of CEM

CEM Research has wide scope of evolution in next few decades. A few techniques have been incorporated to improve the performance of this stream of research. The hybrid GPU–CPU parallelization may acquire considerable attention in few years. The computational time is expected to be reduced by implementing fast convergence methods and implementing efficient preconditioners. Multi scaling methods that are fast and accurate use efficient DDM methods like discontinuous Galerkin methods and generalized transition matrix. Methods with higher order modeling capabilities with high order basis function, characteristic basis functions are expected to have considerable growth. Novel integral and differential methods can be used for several realistic applications and these applications may also require choice of different material in cases like nonlinear sensing, Spectroscopy, frequency generation. The collaboration of CEM with other field of research like Multiphysics aids in dealing with realistic applications and has potential to grow tremendously.

7. Conclusion

A review on different computational electromagnetics shows that these methods are application specific. Selecting a correct method that will afford fast and accurate solutions may be a difficult task. Comparing the performance of FDTD, FEM, MLFMA accelerated MOM will give better insight on CEM. Although FDTD uses SIS for enhancing accuracy, parallelization for reducing CPU time and Novel RK-HO FDTD to enhance both, is time consuming. On the other hand FEM is producing spurious solution that makes it not ideal for Eigen value problem. However MOM accelerated MLFMA is found to be suitable for well-conditioned and error controllable solver. Preconditioners speed the convergence, parallelization makes CPU time usage less, incremental multilevel filling and sparsification reduces memory and MOM elliptic formulation promotes the accuracy, thereby proving efficiency of MOM.

To cater the needs of emerging technology CEM incorporates the technique called hybridisation. Introduction of this technique helps in achieving the efficiency in terms of computational time, memory storage and accuracy. ID algorithm reduces computational issues, ID-MLFMA and MLFMA-ACA reduces multiscale problem, open MP-VALU MLFMA method enhances the CPU time, in turn making MOM hybrid more reliable. On analysing pros and cons of CEM techniques, it could be noticed that in application specific tasks, hybridisation of techniques involving MOM accelerated MLFMA are exhibiting better results in terms of memory, accuracy and computational time.

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