Accuracy and Agreement of FDTD, FEM and Wave Matrix Methods for the Electromagnetic Simulation of Waveguide Polarizers

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ABSTRACT Waveguide devices of polarization processing are key elements of modern dual-polarized antenna systems. Such antenna systems carry out polarization transformation and separation of signals with orthogonal polarizations in satellite communications and radar appliances. Well-known electrodynamic methods are used to simulate performance of such devices. This paper investigates the accuracy and mutual matching of wave matrix method with FDTD and FEM. The wave matrix method is based on the theory of scattering and transmission matrices. The main electromagnetic characteristics of a polarizer were obtained through the elements of these matrices. The main characteristics of a polarizer were compared. They include phase difference of orthogonal polarizations, level of voltage standing wave, ellipticity coefficient and level of cross-polarization isolation. Comparison of electromagnetic characteristics was carried out at the example of a waveguide iris polarizer for the operating frequency range from 13.0 to 14.4 GHz. In addition, the process of optimization of electromagnetic characteristics was carried out by changing the geometric dimensions of a polarizer. Calculated by different theoretical methods characteristics were compared and analyzed. Results, which were obtained by considered methods, are in good agreement.

INDEX TERMS electromagnetic simulation; FDTD; FEM; wave matrix; polarizer; waveguide polarizer; waveguide components; circular polarization.

I. INTRODUCTION

Fast development of new generation telecommunication systems [1–9] contributed to the use of new methods of signal processing. Among these methods, special attention is paid to signal polarization processing. It helps to increase the information capacity of communication channels. Such processing is carried out in modern antenna systems. The key element of such systems is polarization conversion devices.

The most spread are polarization processing devices with longitudinal conducting septums [10–14]. They have a compact design. The disadvantage of such devices is their narrow bandwidth.

Alternative variants of polarizers are the designs with reactive elements in the form of diaphragms [15–29], pins [30–33], and their combinations [34, 35]. The reasons are the wider bandwidth and better electromagnetic performance of such devices.

The publication [15] shows the results of development of waveguide polarizers with 11 diaphragms for the operating frequency range of 5.7–7.7 GHz. The device provides 35 dB polarization isolation and maintains a differential phase shift of 90°±2.0° over the entire operating frequency range. The technique for developing waveguide polarizers with diaphragms for two frequency ranges is described in [16]. The device supports 90°±2.0° phase shift. The developed device provides a polarization isolation of 30 dB. The scientific work [17] presents the design of a polarizer of 6 diaphragms for the frequency range 24-36 GHz. It provides 90°±8.0° differential phase shift and 30 dB polarization isolation. An orthomode converter is often used in dual polarization antenna systems together with waveguide polarizers [18]. Such a device separates two orthogonal linearly polarized signals. Moreover, the architecture of a polarizing device with diaphragms has axial symmetry and two mirror planes of symmetry [19]. This feature improves the quality of polarization. The robots [20, 21] show the results of the development of compact waveguide polarizers with two-sided symmetry. Multivariable optimization is applied to determine the best electromagnetic performance.

An analytical method for analyzing the phase, matching, and polarization characteristics of waveguide polarizers with diaphragms was proposed in [22, 23]. The developed technique is based on the scattering and transmission wave matrices. To test the method, a waveguide polarizer with two in the frequency range from 7.4 to 8.5 GHz was
developed. Its differential phase shift varies in ancestors $90^\circ \pm 8^\circ$. The minimum cross-polarization isolation value is 21.5 dB. In [24–26], polarization devices based on a square waveguide with a different number of diaphragms were developed. The best characteristics were provided by a polarizer with five diaphragms in the working Ku-band. An analytical-numerical method for constructive synthesis of optimal waveguide polarizers with diaphragms was proposed in [27]. For testing, a waveguide polarizer based on three diaphragms was developed. It provides a cross-polarization isolation of no more than 26 dB. The article [28] proposed a method for analyzing waveguide polarizers with diaphragms based on equivalent microwave circuits.

The article [29] shows the possibility of using "optical activity" for constructing polarizers, which arises in the interaction of fields on coupled diaphragms. One of the techniques for designing broadband polarizers was proposed in [30]. In [31, 32], the technique of designing waveguide polarizers with pins was modernized. As a result, the optimal pin length and diameter were obtained. The authors of [33] proposed a new method for designing tunable waveguide polarizers with metal pins. The developed technique uses the theory of scattering and transmission wave matrices.

A method for calculating waveguide polarizers with diaphragms and pins was developed in [34–36]. The proposed method can be widely used to develop new tunable waveguide filters, phase shifters and polarization processing devices.

Axially symmetric waveguide has better structure symmetry and less polarization distortion. An example of such structures are waveguides with four ribs [37]. Coaxial and orthomode converters [38, 39] are based on such structures. In [40], a numerical algorithm for the analysis and optimization of polarization devices with a large bandwidth is presented. Such polarizers consist of symmetrically located rectangular ridges in a coaxial waveguide. A mathematical model of sectoral coaxial-ribbed waveguides was proposed in [41]. In [42], mathematical models were obtained using the method of integral equations. The analysis of the characteristics of eigenwaves of coaxial waveguides with four ribs was carried out in [43]. In [44], the design of the antenna power supply system based on a cylindrical waveguide was presented. Developed system has a built-in structure of a polarization conversion device, which provides a symmetric scheme of circular polarization radiation.

In [45] the design of a compact polarization converter based on a square waveguide with diaphragms is presented. The device provides a phase shift of $180^\circ \pm 2^\circ$ in the frequency band 12.8–14.2 GHz. The main relationship between the electromagnetic parameters of polarizers is considered in [46]. In [47] a methodology for constructing waveguide polarizers on diaphragms is presented, and the concept of designing of dual-band polarizers is considered in [48]. In addition, some publications [49, 50] are devoted to the constructions of broadband polarization processing devices. There are also several designs of polarizers for use in radar systems [51], satellite systems [52–54] and both of them [55]. In [56] a comparative analysis of waveguide polarizers with diaphragms is performed. Different ways of tuning waveguide polarization converters are presented in papers [57–59]. In [60] the results of the development of polarizers with elliptical diaphragms are presented, and in [61] a design of diaphragms in the form of grooves in a waveguide is proposed. Different designs of waveguide polarizers with plates that were integrated into the waveguide substrate are considered in [62–65]. In [66] the characteristics of the wave impedances in a circular and rectangular waveguide are considered, which are of particular interest for the development of waveguide polarization converters and other devices. Different theoretical techniques for the development and analysis of polarizers are considered in [67–70].

The aim of the current study is to compare methods for calculating the characteristics of waveguide devices. For this, a polarizing device with diaphragms was created and its electromagnetic characteristics were optimized for the satellite frequency range of 13.0–14.4 GHz. The differential phase shift of the developed device should not exceed $5^\circ$. In this case, the device must provide good cross-polarization isolation, which must exceed 28 dB.

II. FEATURES OF THE APPLIED WAVE MATRIX MODEL

A three-dimensional model of a polarizer with three diaphragms based on a square guide is shown in Fig. 1.

**FIGURE 1.** Inner structure of the square guide polarizer with 3 diaphragms

Fig. 1 shows the internal structure of a square waveguide polarizer with three diaphragms. Designations of all sizes of a design are shown in Fig. 1. Two outer diaphragms have the same height $h_1$. They are below the central diaphragm at a height of $h_2$ to improve structure’s matching. The thickness of all diaphragms is $w$, and the gaps between them are $L_1$. 
Using the method of equivalent microwave circuits [71, 72], an equivalent network of the waveguide polarizer was obtained. It is demonstrated in Fig. 2.

![FIGURE 2. Equivalent network of a polarizer with 3 diaphragms](image)

For the inductive (L) and capacitive (C) diaphragms in a waveguide, the elements of a scattering matrix of individual components are determined in the following form

$$S_{1L} = \frac{jB_L}{2 - jB_L}, S_{21L} = \frac{2}{2 - jB_L},$$

$$S_{1C} = \frac{-jB_C}{2 + jB_C}, S_{21C} = \frac{2}{2 + jB_C},$$

where $B_L$ and $B_C$ are the reactive conductivities of diaphragms in the waveguide.

Next, we obtain the elements of the transfer matrix for each diaphragm

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}.$$  \hspace{1cm} (3)

These elements can be defined based on previously determined scattering matrix as follows

$$T_{11} = \frac{1}{S_{21}}, T_{12} = -\frac{S_{22}}{S_{21}}, T_{21} = -\frac{S_{22}}{S_{21}},$$

$$T_{22} = \frac{S_{22}S_{22} - S_{23}S_{23}}{S_{21}}.$$  \hspace{1cm} (5)

In used equivalent network $T_1$, $T_5$ stand for the transmission wave matrices of the outer diaphragms, $T_3$ is the transmission wave matrix of the middle diaphragm, and $T_2$, $T_4$ designate the transmission wave matrices of two waveguide sections.

The wave transmission matrix of the diaphragms is determined by the following expression

$$[T_2] = [T_4] = \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix},$$

where $\theta$ is the electric length of a each separate diaphragm.

Similarly, the wave matrices of the transmission waveguide lines are determined by the following formula

$$[T_1] = [T_5] = \begin{bmatrix} T_{11,L1} & T_{12,L1} \\ T_{21,L1} & T_{22,L1} \end{bmatrix};$$

$$[T_3] = \begin{bmatrix} T_{11,L2} & T_{12,L2} \\ T_{21,L2} & T_{22,L2} \end{bmatrix};$$  \hspace{1cm} (7)

where $T_{i,L,j}$ are the elements of the wave matrix of waveguide sections between the adjacent diaphragms.

Each element of the polarizer is described by its own transmission matrix, and the overall matrix is determined by the following formula

$$[T_{\Sigma}] = \prod_{i=1}^{5} [T_i] = \begin{bmatrix} T_{\Sigma_{11}} & T_{\Sigma_{12}} \\ T_{\Sigma_{21}} & T_{\Sigma_{22}} \end{bmatrix},$$

where $T_i$ stands for the wave transmission matrix of each individual element of the equivalent circuit.

Using the connection between the wave scattering and transmission matrices [73, 74], the elements of a scattering matrix of the developed device were calculated based on the obtained total transfer matrix:

$$S_{\Sigma} = \begin{bmatrix} S_{\Sigma_{11}} & S_{\Sigma_{12}} \\ S_{\Sigma_{21}} & S_{\Sigma_{22}} \end{bmatrix} = \frac{1}{T_{\Sigma_{11}}} \begin{bmatrix} T_{\Sigma_{11}} & [T] \\ T_{\Sigma_{21}} & 1 - T_{\Sigma_{12}} \end{bmatrix},$$

where $[T]$ is determinant of the matrix from expression (8).

The main electromagnetic characteristics of the mathematical model of the microwave polarizer were expressed through the elements of the obtained scattering wave matrix [75, 76].

The basic electromagnetic characteristics are phase, matching and polarization. Among them, it is necessary to highlight the phase differential, the maximum level of voltage standing wave (LVSW) for vertical and horizontal polarizations, the maximum level of the ellipticity factor, the minimum level of cross-polarization isolation (LCI).

At the first stage of the polarization device optimization process, the initial values of the geometric dimensions are determined by the method [73]. Since the device under development must operate in the operating frequency range of 13.0-14.4 GHz, the size of the square waveguide wall will vary from 14 mm to 29 mm. The center frequency of the band will be 11.75 GHz.

The optimization process is carried out according to the following technique from [27]. At the initial stage, due to a change in the size of the waveguide wall, we achieve the value of the derivative of the differential phase with respect to frequency equal to zero. Then, after a set of changes in the heights of the diaphragms, we achieve that the value of the differential phase shift was as close as possible to 90°. Then, by changing the distance between the diaphragms, we achieve the minimum LVSW of vertical and horizontal polarizations.

### III. RESULTS OF ANALYSIS OF ELECTROMAGNETIC CHARACTERISTICS OF A WAVEGUIDE POLARIZER

The section presents the results of numerical modeling of the main electromagnetic characteristics of the developed polarization conversion device.

To check the correctness of the results obtained, let us compare the electromagnetic characteristics that were obtained by different methods. Well-known electromagnetic...
methods were chosen as methods, such as the method of finite integration in the time domain and the method of finite elements in the private domain [77–81]. In Fig. 3 shows the frequency dependence of the differential phase shift of the optimized polarizer on frequency in the Ku-band for three methods.

We emphasize that the differential phase shift takes the values: 90°±4°, 90°±5° and 90°±3° for the proposed method, the finite integration method and the finite element method in the entire operating frequency range, respectively. It should be noted that all three methods give the same result with small deviations.

In Fig. 4 shows the dependence of the LVSW of the optimized polarization device on frequency for horizontal polarization in the operating range of 13.0–14.4 GHz for three methods. It should be noted that the maximum LVSW value is reached at 14.4 GHz. In this case, the maximum LVSW level takes on the values 1.96, 2.06, 2.1 for the proposed method, the finite integration method and the finite element method in the entire operating frequency range, respectively.

Therefore, for two polarizations, the maximum LVSW values are determined from the highest values for vertical and horizontal polarizations. In this case, the maximum LVSW takes on the values 2.04, 2.06, 2.2 for the proposed method, the finite integration method and the finite element method in the entire operating frequency range, respectively.

It should be noted that the maximum LVSW value is reached at 14.4 GHz. In this case, the maximum LVSW level takes on the values 1.96, 2.06, 2.1 for the proposed method, the finite integration method and the finite element method in the entire operating frequency range, respectively.

Fig. 6 shows the dependence of the ellipticity coefficient on the frequency of the optimized polarizer for
the proposed method, the finite integration method and the finite element method over the entire operating frequency range, respectively.

![Image](image1.png)

**FIGURE 6.** Frequency dependence of the ellipticity coefficient

Note that in the frequency range from 13.0 GHz to 14.4 GHz, the ellipticity coefficient of the developed polarization device takes on such peak values of 0.6 dB, 0.5 dB, 0.7 dB for the proposed method, the finite integration method and the finite element method, respectively.

The dependence of level of crosspolar isolation on frequency for the developed device for polarization conversion is shown in Fig. 7 for the proposed method, finite integration method and finite element method over the entire operating frequency range, respectively.

We emphasize that in the frequency range from 13.0 GHz to 14.4 GHz, the minimum value of the crosspolarization isolation of the developed polarization device corresponds to such values of 29.5 dB, 30.5 dB, 27.4 dB for the proposed method, the finite integration method and the finite element method, respectively.

![Image](image2.png)

**FIGURE 7.** Frequency dependence the LCI for a polarizing device

It can be seen from the Fig. 8 that the minimum return loss value is 9 dB for vertical and horizontal polarizations. The maximum return loss value is 44 dB.

As a result, in the frequency range from 13.0 GHz to 14.4 GHz, the developed waveguide polarization device with three diaphragms maintains the following necessary characteristics. The phase differential of the device is in the range of 90° ± 4.0°. In this case, the maximum LVSW level for vertical and horizontal polarizations is 2.04. The peak ellipticity factor is 0.6 dB and the minimum LCI is 29.5 dB.

**IV. ANALYSIS OF OPTIMIZATION RESULTS**

All geometrical dimensions of the developed waveguide polarizing device with three diaphragms are presented in Tables 1. These dimensions of the proposed device were obtained as a result of the optimization procedure for the operating frequency range of 13.0–14.4 GHz.

For the obtained dimensions of the developed polarization device, the basic electromagnetic characteristics in the operating frequency range of 13.0-14.4 GHz were calculated and optimized using the proposed method [82–85], the finite integration method [86] and the finite element method [87, 88], respectively, which are presented in Tables II-IV.

**TABLE I. GEOMETRICAL DIMENSIONS OF THE DEVELOPED POLARIZATION DEVICE FOR THE OPERATING FREQUENCY RANGE OF 13.0-14.4 GHz**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The size of the wall of a square waveguide</td>
<td>21.4 mm</td>
</tr>
<tr>
<td>Middle diaphragm height</td>
<td>3.82 mm</td>
</tr>
</tbody>
</table>
For the obtained dimensions of the developed polarization device, the basic electromagnetic characteristics in the operating frequency range of 13.0-14.4 GHz were calculated and optimized using the proposed method [82–85], the finite integration method [86] and the finite element method [87–89], respectively, which are presented in Tables II-IV.

### Table II. Electromagnetic Characteristics of the Polarization Device Obtained by the Proposed Methods for the Operating Frequency Range of 13.0-14.4 GHz

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of diaphragms</td>
<td>2.39 mm</td>
</tr>
<tr>
<td>Distance between diaphragms</td>
<td>4.87 mm</td>
</tr>
<tr>
<td>Thickness of all diaphragms</td>
<td>2.75 mm</td>
</tr>
</tbody>
</table>

### Table III. Electromagnetic Characteristics of the Polarization Device Obtained by the Finite Integration Methods for the Operating Frequency Range of 13.0-14.4 GHz

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase difference</td>
<td>90° ± 4.0°</td>
</tr>
<tr>
<td>Maximum LVSW</td>
<td>2.04</td>
</tr>
<tr>
<td>Ellipticity coefficient</td>
<td>0.6 dB</td>
</tr>
<tr>
<td>Minimum LCI</td>
<td>29.5 dB</td>
</tr>
</tbody>
</table>

### Table IV. Electromagnetic Characteristics of the Polarization Device Obtained by the Finite Element Methods for the Operating Frequency Range of 13.0-14.4 GHz

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase difference</td>
<td>90° ± 3.0°</td>
</tr>
<tr>
<td>Maximum LVSW</td>
<td>2.06</td>
</tr>
<tr>
<td>Ellipticity coefficient</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>Minimum LCI</td>
<td>30.5 dB</td>
</tr>
</tbody>
</table>

Table V shows that the main electromagnetic characteristics of analogs correlate with the same characteristics of the developed device. It should be noted that a smaller value of the deviation of the differential phase shift from 90° can be achieved by increasing the number of diaphragms inside the polarizer. This, in turn, leads to lengthening of the structure and an increase in its size. When developing satellite antenna systems, it is often required to design small-sized and lightweight devices and assemblies. In terms of overall dimensions and characteristics, a developed waveguide polarizer meets the requirements of modern radio engineering systems.

### Table V. Comparison of Main Electromagnetic Characteristics of Waveguide Polarizers

<table>
<thead>
<tr>
<th>Number of analogue’s reference</th>
<th>Phase difference of polarizations</th>
<th>LCI, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>90° ± 10.0°</td>
<td>30.0 dB</td>
</tr>
<tr>
<td>12</td>
<td>90° ± 2.0°</td>
<td>35.0 dB</td>
</tr>
<tr>
<td>13</td>
<td>90° ± 2.0°</td>
<td>30.0 dB</td>
</tr>
<tr>
<td>14</td>
<td>90° ± 8.0°</td>
<td>30.0 dB</td>
</tr>
<tr>
<td>16</td>
<td>90° ± 2.5°</td>
<td>33.0 dB</td>
</tr>
<tr>
<td>19</td>
<td>90° ± 8.0°</td>
<td>21.5 dB</td>
</tr>
<tr>
<td>20</td>
<td>90° ± 4.4°</td>
<td>27.5 dB</td>
</tr>
<tr>
<td>25</td>
<td>90° ± 4.0°</td>
<td>28.0 dB</td>
</tr>
<tr>
<td>26</td>
<td>90° ± 4.2°</td>
<td>28.4 dB</td>
</tr>
<tr>
<td>31</td>
<td>90° ± 5.4°</td>
<td>26.5 dB</td>
</tr>
<tr>
<td>32</td>
<td>90° ± 2.2°</td>
<td>29.0 dB</td>
</tr>
<tr>
<td>34</td>
<td>90° ± 3.2°</td>
<td>31.0 dB</td>
</tr>
</tbody>
</table>

Tables 2–4 show that the developed device for polarization conversion with three diaphragms provides satisfactory matching in telecommunication systems. Note that the difference in the maximum deviation of the phase differential value for all methods does not exceed 1°. In addition, the LVSW value obtained by the three methods does not differ by more than 1.4. The difference between the maximum values of the ellipticity factor for all three methods is 0.1 dB. Therefore, the designed triple diaphragm polarization converter maintains satisfactory matching, good phase and polarization characteristics over the entire operating frequency range of 13.0-14.4 GHz.

Let us compare the main electromagnetic characteristics of the developed polarization device with the characteristics of analogs, which are shown in Table V. As parameters for comparison, we will choose the differential phase shift and cross-polarization decoupling.

### V. Conclusions

The paper compares the accuracy and matching of existent theoretical methods of waveguide devices analysis at the example of a guide polarizer with diaphragms. It was developed and optimized for satellite antenna systems in the frequency range 13.0–14.4 GHz.

The electromagnetic characteristics of the polarizer were obtained using the FDTD, FEM and Wave Matrix Method. The polarizer provides differential phase shift of 90° ± 3.0°, 90° ± 5.0°, 90° ± 4.0° for these three methods, respectively. The phase differential for orthogonally polarized modes of the polarizer, which was obtained by these methods, agree with the accuracy of 2°. The polarizer provides LVSW of 2.06, 2.2, 2.04 for the three methods,
respectively. The LVSW of the polarizer obtained by these methods is in agreement with an accuracy of 0.16. The developed polarizer provides an ellipticity factor of 0.5 dB, 0.7 dB, 0.6 dB for the three methods, respectively. The ellipticity coefficient of the polarizer obtained by the methods is consistent with an accuracy of 0.2. The polarizer provides LCI of 30.5 dB, 27.4 dB, 29.5 dB for the FDTD, FEM and Wave Matrix Method, respectively. The LCI of the polarizer obtained by these methods agree with an accuracy of 3.1 dB.

Therefore, both numerical FDTD and FEM techniques and theoretical Wave Matrix Method provide high accuracy of the results and can be effectively applied for the development and optimization of waveguide devices.

REFERENCES


