

# An Efficient Approximate Method for Scattering Response from Infinite Arrays of Nonlinearly Loaded Antenna in the Frequency Domain

Saeed Reza Ostadzadeh<sup>1,2</sup>, Mohaddeseh Salehi<sup>3</sup> and Bahareh Jafari<sup>4</sup>

<sup>1</sup>Faculty of Engineering, Arak University, Arak, Iran.

<sup>2</sup>Research Institute of Renewable Energy, Arak University, Arak, 38156-88349, Iran.

<sup>3,4</sup>BSC students, Department of Engineering, Arak University, Arak, Iran.

Corresponding author: Saeed Reza Ostadzadeh (e-mail: s-ostadzadeh@araku.ac.ir).

**ABSTRACT** In this paper, different arrangements of infinite arrays of nonlinearly loaded antennas are analyzed in the frequency domain by an efficient approximate method and compared with the exact one which are respectively based on the nonlinear current and harmonic balance techniques. In one hand, although the exact method is suitable for strongly nonlinear load, it is suffering from gradient operation and initial guess in the iteration process especially under multi-tone excitations. On the other hand, although the approximate method is very efficient, it is limited to weakly nonlinear loads and low-valued incident waves. Finally, acceptable ranges for application of the approximate method versus different parameters such as nonlinearity effect of the load and the magnitude of incident wave are extracted.

**INDEX TERMS** Infinite array, harmonic balance, nonlinear current, and nonlinear load.

## I. INTRODUCTION

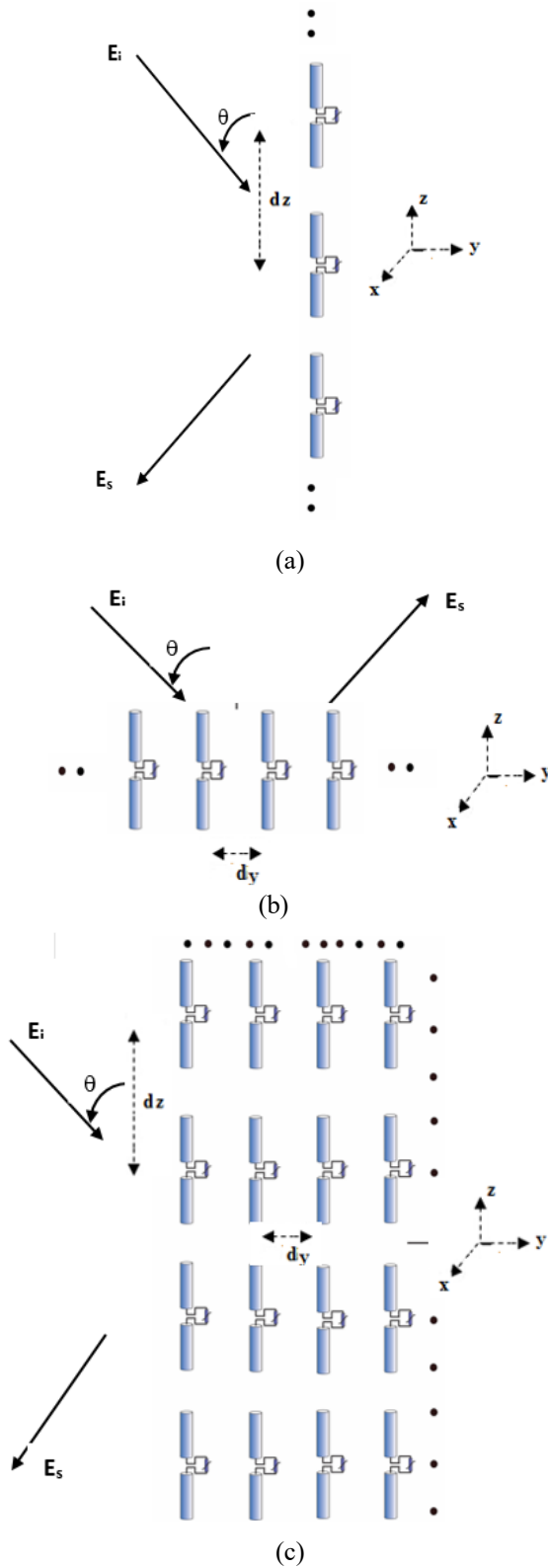
Infinite array of nonlinearly loaded antennas can be used as phase conjugation in the frequency domain or time reversal in the time domain [1, 2]. Fig. 1 shows different arrangements of infinite array of straight wire antennas including parallel, collinear and planar structures which are centrally terminated with nonlinear loads. Such structures are called nonlinear antennas for simplicity.

In general, analyzing such structures includes exact and approximate methods. Both methods solve the microwave equivalent circuit as shown in Fig. 2 somehow. Analysis in Fig. 2 consists of two parts. The first part is Norton's equivalent circuit and computed based on the method of moments (MoM) [3] or efficient qualitative methods [4-6] and the second part is a nonlinear load. The main goal in such structures is computation of the induced voltage across the nonlinear load at different harmonic frequencies based on the exact [7-13] and approximate methods [14-16].

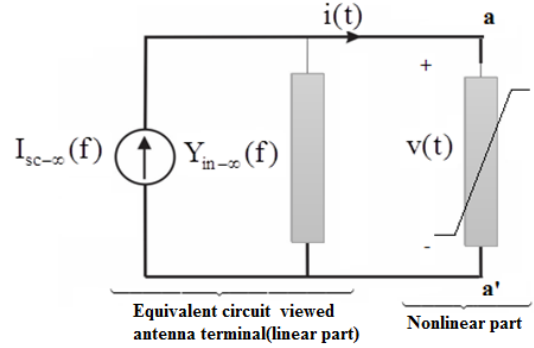
Exact methods such as harmonic balance (HB) technique [7] can be used for arbitrarily nonlinear load. However, it suffers from complex computations especially for narrow-band electromagnetic pulses and in array structures. To remove this drawback, different optimization techniques [17-21] have been used, but they still require initial guess which sometimes lead to local solutions. Therefore, the efficient intelligent methods such as neural networks [22] and fuzzy inference systems [23] were later proposed. However, they

need too many input-output pairs which should be computed by HB technique in advance.

Approximate methods, on the other hand, including Volterra series [14, 15] and nonlinear currents techniques [16] are very efficient, but they are limited to weakly nonlinear loads. Between the two approximate methods, the one based on nonlinear current technique (NC) is more efficient since each frequency component of the induced voltage is separately computed. This property is very advantageous in the design of such structures to control the scattering response from nonlinear antennas at desired frequency component.



**FIGURE 1.** Different infinite arrays of nonlinearly loaded straight wire antennas arranged in (a): collinear, (b): parallel, and (c): planar structures.



**FIGURE 2.** Equivalent circuit of different infinite arrays of nonlinear antennas consisting of linear and nonlinear parts.

To the best of our knowledge, this efficient method has been used for analyzing single and finite array of nonlinear antenna [16]. Its application, however, on the different infinite arrays of nonlinear antennas has not been yet investigated. This motivates the authors to validate this approximate method on different infinite arrays of nonlinear antennas under single and multi-tone excitations and its applicable ranges are extracted.

This paper is organized as follows. In section II, modeling principles of exact and approximate methods are briefly explained. Section III is focused on the validity of the approximate method in comparison with existing methods in literatures. In section IV, numerical results based on the approximate method for different arrangements of infinite arrays is presented. Sensitivity analyses and extracting acceptable ranges for the problem parameters are carried out in section V. Finally, concluding remarks are given in section VI.

## II. ANALYSIS OF INFINITE ARRAY OF NONLINEAR WIRE ANTENNAS

As mentioned in the previous section, analysis of infinite array of nonlinear antennas consists of linear and nonlinear parts. In this section, analyses of the two parts are separately explained briefly.

### A. LINEAR PART

Two quantities in the linear part of Fig. 2, namely input admittance ( $Y_{in-\infty}$ ) and short circuit current ( $I_{sc-\infty}$ ) are computed via applying the MoM on the following integral equation [24]

$$\hat{t} \cdot \bar{E}_i = \frac{j\omega\mu}{4\pi} \int_{\text{antenna}} \bar{I}_i(\bar{r}') \cdot G_{\infty}(\bar{r}, \bar{r}') d\bar{l} \quad (1)$$

where  $\hat{t}$  is the unit tangential vector along wire antenna length,  $\bar{E}_i$  is the incident electric field, and  $\bar{I}_i(\bar{r}')$  is the induced current along the antenna that is unknown. Also,  $G_{\infty}(\bar{r}, \bar{r}')$  denotes the Green's function of infinite array for the electric field at distance vector  $\bar{r}$  due to a current at distance vector  $\bar{r}'$  [25] as below

$$G_{\infty} = \sum_{q=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} G_0 \quad (2)$$

In (2),  $G_0$  is the Green's function of isolated wire antenna and computed as below

$$G_0 = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{-2jk_x} e^{jk_x x} e^{jk_z(z-pd_z)} e^{jk_y[y-(qd_y+y')] } dk_z dk_y, \quad (3)$$

$$-L/2 < y' < L/2$$

Where

$$k_x = \begin{cases} \sqrt{k^2 - k_z^2 - k_y^2} & k^2 \geq (k_z^2 + k_y^2) \\ -j\sqrt{k^2 - k_z^2 - k_y^2} & k^2 < (k_z^2 + k_y^2) \end{cases} \quad (4)$$

Also,  $k$  is wave number, and  $d_y, d_z$  are the spacings between antennas in  $y$  and  $z$ -axis directions as shown in Fig. 1(c). Now, numerical solution of Eq. (1) based on the MoM, the current distribution along the wire antenna is then determined. Once it is computed at different harmonic frequencies,  $Y_{in-\infty}$  and  $I_{sc-\infty}$  can be accordingly determined.

## B. NONLINEAR PART

In this section, computational principals of exact and approximate methods for computing the induced voltage across the nonlinear load of Fig. 2 are explained in detail.

### 1) HARMONIC BALANCE TECHNIQUE

According to [8], HB-based analysis of infinite array of nonlinear antennas is based on numerical solution of the microwave equivalent circuit of Fig. 2. In this analysis, at first Kirchhoff's Voltage Law (KCL) on the node "a" of Fig. 2 is applied and an error function vector  $\bar{\varepsilon}$  [8] is then achieved, that is

$$\bar{\varepsilon} = \bar{Y}_{in-\infty} \bar{V}_s - \bar{I}_{sc-\infty} + \bar{Df}(\bar{T}\bar{V}_s) \rightarrow \bar{0} \quad (6)$$

All quantities in (6) are defined as follows:

$\bar{Y}_{in-\infty}$  is the matrix of input admittance of the center antenna terminal of the infinite array at mixing frequencies, i.e.

$$\bar{Y}_{in-\infty} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & G_{lin\infty} & B_{lin\infty} & 0 & \cdots & 0 \\ 0 & -B_{lin\infty} & G_{lin\infty} & \ddots & 0 & \vdots \\ \vdots & \vdots & \vdots & 0 & G_{Nin\infty} & B_{Nin\infty} \\ 0 & & & -B_{Nin\infty} & G_{Nin\infty} & 0 \end{bmatrix} \quad (7)$$

Where  $G_{kin\infty}$  and  $B_{kin\infty}$  are respectively real and imaginary parts of input admittance of infinite array at  $k$ th mixing frequency.

$\bar{I}_{sc-\infty}$  is short circuit current vector at antenna terminal and computed only at exciting frequencies. However,  $\bar{V}_s$  is an unknown vector of induced voltage across the nonlinear load at mixing frequencies, that is

$$\bar{V}_s = [V_{s,0} \quad V_{s,1} \quad V_{s,2} \quad \cdots \quad V_{s,2N-1} \quad V_{s,2N}]^T \quad (8)$$

$\bar{D}$  and  $\bar{T}$  are respectively conversion matrices from the frequency domain to time domain and vice versa. In addition,  $f(\cdot)$  is the (i-v) characteristics of the nonlinear load and  $N$  is an integer denoting the number of mixing frequencies.

Finally, the unknown vector  $\bar{V}_s$  in (6) is computed in an iteration process using Newton-Raphson algorithm as below

$$\bar{V}_{s,i+1} = \bar{V}_{s,i} - \bar{J}\bar{\varepsilon}, \quad i = 1, 2, \dots \quad (9)$$

Where  $\bar{J}$  is Jacobin and defined as derivative of  $\bar{\varepsilon}$  with respect to  $\bar{V}_s$ .

### 2) NONLINEAR CURRENT TECHNIQUE

In this section, the approximate method based on nonlinear current technique is applied on the infinite arrays of nonlinear antennas. In dealing with such structures based on NC technique, the nonlinear load is first represented as the following power series

$$i = g_1 v + g_2 v^2 + g_3 v^3 + \dots \quad (10)$$

The voltage in (10) can be expressed as

$$v(t) = v^1(t) + v^2(t) + v^3(t) + \dots \quad (11)$$

Where  $v^{(k)}(t)$  represents the sum of all mixing frequency components of  $k$ th order. If the voltage across the nonlinear load is limited to third order, the current  $i(t)$  in (10) can be divided into linear term  $i_{linear}(t)$  and nonlinear term  $i_{nonlinear}(t)$  as below

$$i_{linear}(t) = g_1 v(t) \quad (12)$$

$$i_{nonlinear}(t) = g_2 v^2(t) + g_3 v^3(t) \quad (13)$$

The linear part in (12) represents a linear resistor with resistance of  $1/g_1$ . If the nonlinearity of the load is limited to third-degree then the mixing frequency components of third order are as below

$$i_{nonlinear}(t) = \{g_1 v^{(1)}(t)^2\} + \{2g_2[v^{(1)}(t)][v^{(2)}(t)] + g_3[v^{(1)}(t)]^3\} \quad (14)$$

The above equation can be divided into  $i^{(2)}(t)$ , and  $i^{(3)}(t)$ , i.e.

$$i_{nonlinear}(t) = i^{(2)}(t) + i^{(3)}(t) \quad (15)$$

Where

$$i^{(2)}(t) = g_2[v^{(1)}(t)]^2 \quad (16)$$

$$i^{(3)}(t) = 2g_2[v^{(1)}(t)][v^{(2)}(t)] + g_3[v^{(1)}(t)]^3 \quad (17)$$

According to (15), (16) and (17), the nonlinear equivalent circuit in Fig. 2 can be redrawn as shown in Fig. 3. The

goal of the analysis is to obtain the terminal voltage of the center antenna of the infinite array. Using the substitution theorem and setting all the current sources except  $i_{sc}(t)$  to be zero initially, i.e.,  $i^{(2)}(t) = 0$  and  $i^{(3)}(t) = 0$ , one can obtain the voltage component from the contribution of  $i_{sc}(t)$  only. This voltage component is regarded as  $v^{(1)}(t)$  in (16) and (17). The total iteration sequence is given as

$$v^{(1)} \rightarrow i^{(2)}(t) \rightarrow v^{(2)}(t) \rightarrow i^{(3)}(t) \rightarrow v^{(3)}(t) \quad (18)$$

linear  $\Leftarrow$  nonlinear part (method of nonlinear currents)

For instance, assume that we have a multi-tone excitation

$$i_s(t) = \sum_{q=-Q}^{+Q} I_q^s e^{j\omega_q t} \quad (19)$$

By setting all the current sources except  $i_{sc}(t)$  to be zero initially, one can obtain

$$v^{(1)}(t) = \sum_{q=-Q}^{+Q} V_q^{(1)} e^{j\omega_q t} \quad (20)$$

Where  $V_q^{(1)} = I_q^s / (Y_{in}(\omega_q) + g_1)$ . From (20), the current source  $i^{(2)}(t)$  is computed and then setting all current sources except  $i^{(2)}(t)$  to be zero, one can obtain

$$v^{(2)}(t) = \sum_{q_1=-Q}^{+Q} \sum_{q_2=-Q}^{+Q} \frac{g_2 V_{q_1}^{(1)} V_{q_2}^{(2)}}{Y_{in}(\omega_{q_1} + \omega_{q_2}) + g_1} e^{j(\omega_{q_1} + \omega_{q_2})t} \quad (21)$$

Similarly from (21), the current source  $i^{(3)}(t)$  and accordingly the third-order voltage caused by  $v^{(3)}(t)$  is then given in (22).

$$v^{(3)}(t) = 2g_2 \sum_{q_1=-Q}^{+Q} \sum_{q_2=-Q}^{+Q} \sum_{q_3=-Q}^{+Q} \frac{g_2 V_{q_1}^{(1)} V_{q_2}^{(2)} V_{q_3}^{(3)}}{Y_{in}(\omega_{q_1} + \omega_{q_2} + \omega_{q_3}) + g_1} \times e^{j(\omega_{q_1} + \omega_{q_2} + \omega_{q_3})t} - \frac{Y_{in}(\omega_{q_1} + \omega_{q_2} + \omega_{q_3}) + g_1}{g_3 \sum_{q_1=-Q}^{+Q} \sum_{q_2=-Q}^{+Q} \sum_{q_3=-Q}^{+Q} \frac{V_{q_1}^{(1)} V_{q_2}^{(2)} V_{q_3}^{(3)}}{Y_{in}(\omega_{q_1} + \omega_{q_2} + \omega_{q_3}) + g_1} \times e^{j(\omega_{q_1} + \omega_{q_2} + \omega_{q_3})t}} \quad (22)$$

Finally, the total voltage response  $v(t)$  can be found from (11), (20), (21), and (22). As seen in (11), (20), (21) and (22), in despite of Volterra series [15], efficient closed-form expression for induced voltage at  $k$ th order is easily computed without needing to compute the total response.

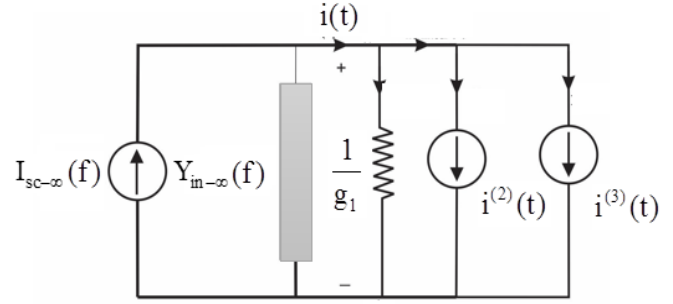


FIGURE 3. Equivalent circuit of figure 2 based on the NC technique.

### III. VALIDITY

In this section, validity of the approximate method for two examples of infinite planar array of nonlinear antennas is investigated [8, 22]. For both examples, the array is normally illuminated by an incident plane wave.

In the first example, the exciting wave has single frequency 150 MHz and magnitude of 1V/m. The spacing between antennas in y and z- directions (dy and dz) are 2m. Also each antenna in the array is centrally terminated with a Gun diode with the following (i-v) characteristic:

$$i = 1/75v + 4v^3 \quad (23)$$

The simulation results at the three mixing frequencies, i.e., 150, 300, and 450 MHz, using the approximate method and the neural networks based on the radial basis functions (NN-RBF) [22] are shown in Fig. 4. As can be seen, good agreement is achieved.

In the second example, the previous planar array is again used, but the exciting wave has two frequencies 140 MHz and 160 MHz with magnitude of 0.1V/m. Moreover, the nonlinear load is a p-n diode with the (i-v) characteristic as expressed in (24):

$$i = I_s (e^{v/v_T} - 1) \quad (24)$$

Where  $I_s = 10\text{nA}$ , and  $v_T = 26\text{mV}$ . The dc-biased circuit for the diode is shown in Fig. 5. The simulation results at different mixing frequencies by the approximate method in this paper and inexact newton algorithm (INA) [8] are shown in Fig. 6. From this figure, slight error between the two methods is observed which is acceptable in electromagnetic engineering.

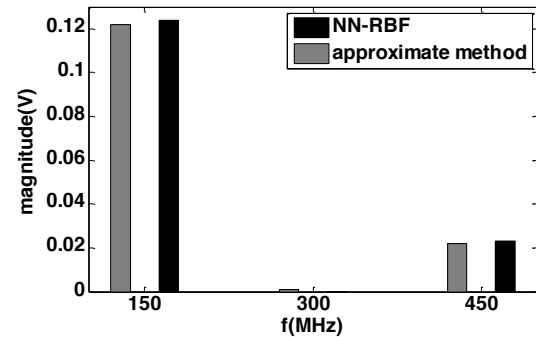
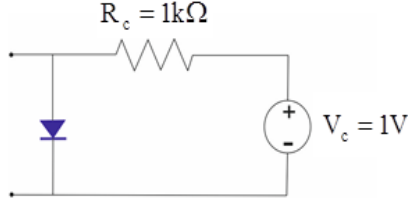
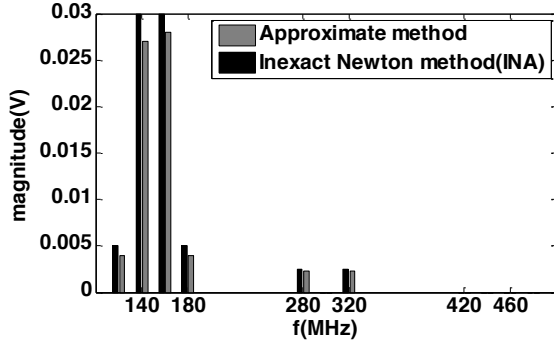


FIGURE 4. Magnitude of induced voltage across the Gun diode based on the NN-RBF [22] and the approximate method in this paper.



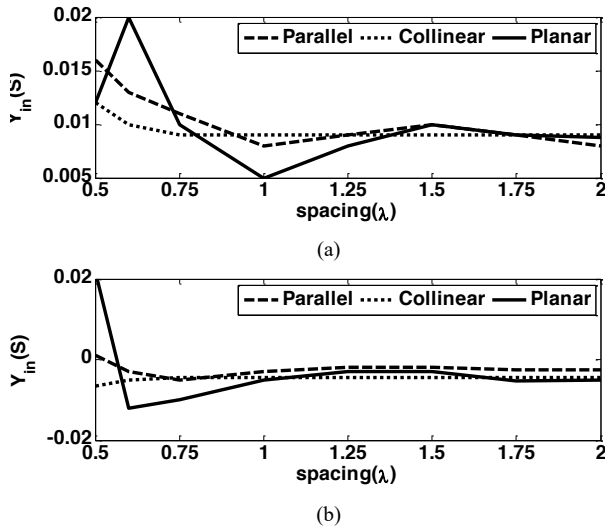
**FIGURE 5.** The circuit of the dc-biased diode connected to the center of each antenna in the infinite arrays.



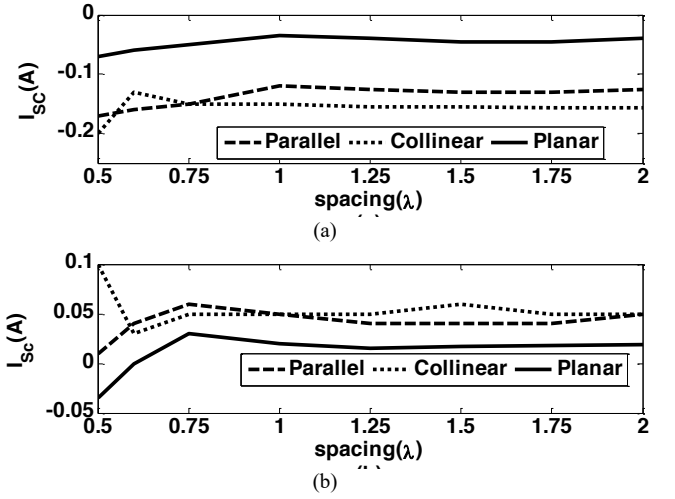
**FIGURE 6.** Magnitude of induced voltage across the p-n diode based on the INA [8] and the approximate method in this paper.

#### IV. NUMERICAL RESULTS

In this section, the input admittance ( $Y_{in-\infty}$ ), and short circuit current ( $I_{sc-\infty}$ ) for three different arrangements of infinite array versus normalized spacings ( $d_y, d_z/\lambda$ ) are first computed based on the MoM and shown in Figs. 7 and 8 respectively. In these figures, the vertical and horizontal spacings between antennas are assumed to be the same. After then, nonlinear analyses under single and multi-tone excitations are carried out.



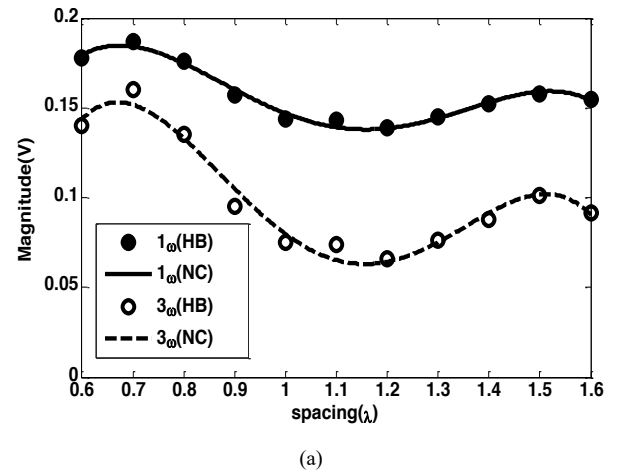
**FIGURE 7.** Input admittance (S) of different arrangements of infinite array of straight wire antennas versus normalized spacing based on MoM, (a): real and (b): imaginary parts.

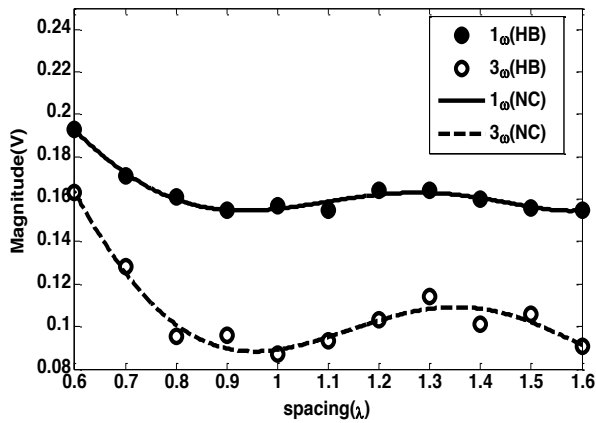


**FIGURE 8.** Short circuit current (A) of different arrangements of infinite array of straight wire antennas versus normalized spacing based on MoM, (a): real and (b): imaginary parts.

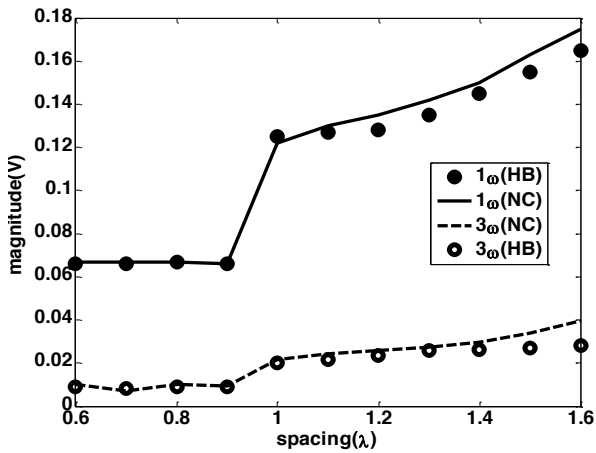
#### A. SINGLE-TONE EXCITATION

Now the approximate method (NC) is applied to different infinite arrays of nonlinear antennas under single tone excitation and its validity in comparison with harmonic balance technique (HB) is presented. The nonlinear load is a Gun diode [7] as expressed in (23). The antennas in the array are straight wire antennas normally illuminated by an incident plane wave of magnitude  $E_i = 1V/m$  including exciting frequency of 150 MHz. The distances among antennas in vertical and horizontal directions are the same. The induced voltages based on the two methods at two mixing harmonic frequencies ( $1\omega, 3\omega$ ) versus normalized spacing are shown in Fig. 8. As can be seen, good agreement is achieved. Note that due to nonlinearity of the load, the induced voltage at second order is zero since  $g_2 = 0$  (see Eq. (21)) and thus not shown in Fig. 9.





(b)

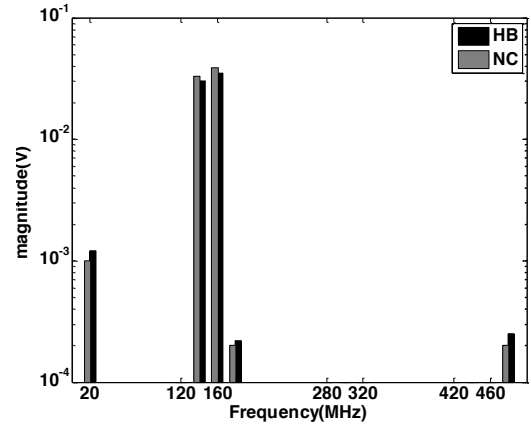


(c)

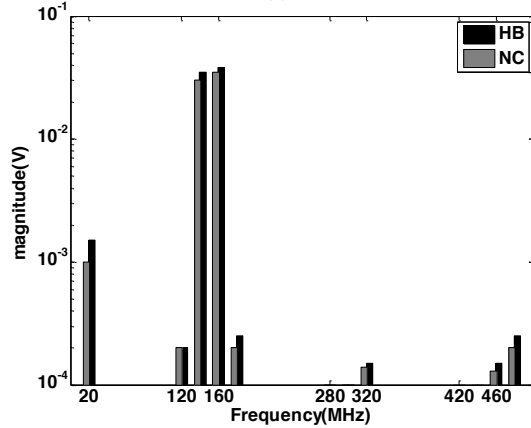
**FIGURE 9.** Magnitude of induced voltage across the Gun diode under single-tone excitation by NC and HB techniques in (a): parallel, (b): collinear, and (c): planar arrangements.

### B. MULTI-TONE EXCITATION

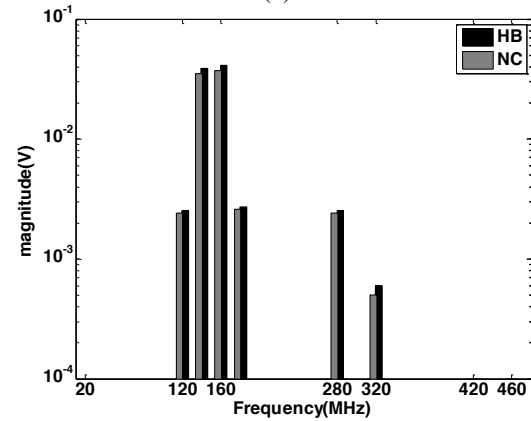
In the case of double-tone excitation, the infinite arrays in the previous sub-section are again used, but the exciting waves having two exciting frequencies  $f_1 = 140\text{MHz}$  and  $f_2 = 160\text{MHz}$  with magnitudes of  $E_{i1}, E_{i2} = 0.1\text{V/m}$ . The nonlinear load is a p-n diode as expressed in (24). The spacings among antennas in vertical and horizontal directions are  $1\text{m}$ .



(a)



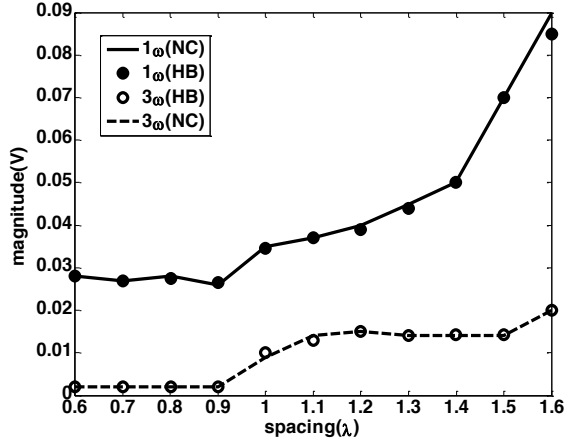
(b)



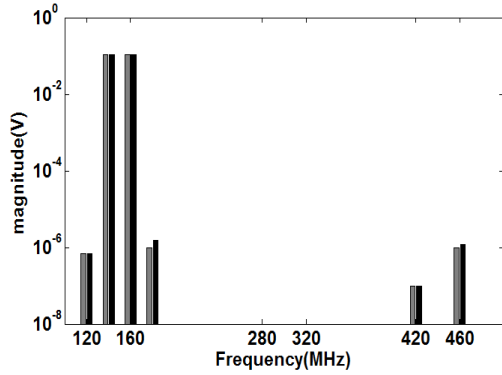
(c)

**FIGURE 10.** Magnitude of induced voltage across the p-n junction diode under double-tone excitation based on the NC and HB techniques in (a): parallel, (b): collinear, and (c): planar arrangements.

The induced voltage at mixing frequencies are computed via the exact and approximate methods and shown in Fig. 10. As can be seen, good agreement between the two methods is achieved. Besides, the frequency selective property at fundamental frequencies, i.e., 140 MHz, 160 MHz, is approximately achieved.



**FIGURE 11.** Magnitude of induced voltage across the Gun diode under oblique incidence and terminated with Gun diode by NC and HB techniques.



**FIGURE 12.** Magnitude of induced voltage across the silicon diode expressed by (25) in infinite collinear array of nonlinear antennas under double-tone and normal incidence by NC and HB techniques.

Evidently, the maximum value of relative error due to the approximate method becomes less under single-tone and oblique incidence of exciting waves and more weakly nonlinear loads. Fig. 11 shows this fact for planar infinite array terminated with Gun diode under single-tone and oblique incidence with  $\theta = 45^\circ$ . Comparing Figs. 9(c) and 11 proves this fact. Moreover, Fig. 12 shows the magnitude of induced voltage in the collinear array illuminated by a double-tone and normal incidence with magnitude of  $E_i = 1\text{V/m}$ . Each antenna in the collinear array is terminated with a more weakly nonlinear load namely silicon diode [26] with the following (i-v) characteristic  $i = v + 0.15v^3$  (25)

### C. RUN TIME

As a final advantageous of the approximate method (NC) against the exact method (HB), the computation times of the induced voltage based on the two methods are compared. Prior to comparison, it should be noted that although the Norton's equivalent circuit parameters as shown in Figs. 3 and 5 are different for the three infinite arrays, the computation principles of induced voltage based on the NC method, i.e., closed-form expressions of (20), (21), and (22),

are the same for each infinite array. Therefore, the NC-based computation times for the three infinite arrays are expected to be same. In the case of HB method, however, the computation of induced voltage is based on minimizing the cost function in Eq. (6) which is dependent on the Norton's equivalent circuit parameters. It means that three cost functions should be separately solved which different computation times for the three infinite arrays are inferred. The comparisons are numerically listed in Tables. I, II, and III respectively for the infinite collinear, parallel, and planar arrays of nonlinear antennas. In these tables, single, double, triple and quadruple-tone excitations are respectively denoted by 'S', 'D', 'T', and 'Q'. Moreover, in despite of HB method, the run time of the NC method is vanishingly short. This fact is more pronounced when the number of exciting frequencies increased.

**TABLE I.** Run time (sec) of HB and NC techniques for infinite collinear array of nonlinear antennas.

| Arrangement | Collinear |      |      |      |
|-------------|-----------|------|------|------|
| Excitation  | S         | D    | T    | Q    |
| NC          | 0.12      | 0.14 | 0.17 | 0.25 |
| HB          | 62        | 113  | 163  | 320  |

**TABLE II.** Run time (sec) of HB and NC techniques for infinite parallel array of nonlinear antennas.

| Arrangement | Parallel |      |      |      |
|-------------|----------|------|------|------|
| Excitation  | S        | D    | T    | Q    |
| NC          | 0.12     | 0.14 | 0.17 | 0.25 |
| HB          | 59       | 120  | 160  | 311  |

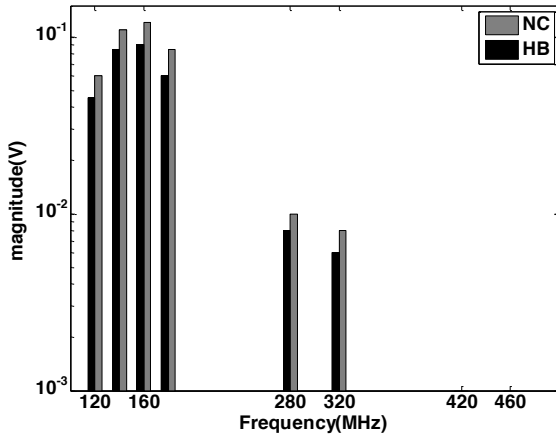
**TABLE III.** Run time (sec) of HB and NC techniques for infinite planar array of nonlinear antennas.

| Arrangement | Planar |      |      |      |
|-------------|--------|------|------|------|
| Excitation  | S      | D    | T    | Q    |
| NC          | 0.12   | 0.14 | 0.17 | 0.25 |
| HB          | 65     | 110  | 170  | 334  |

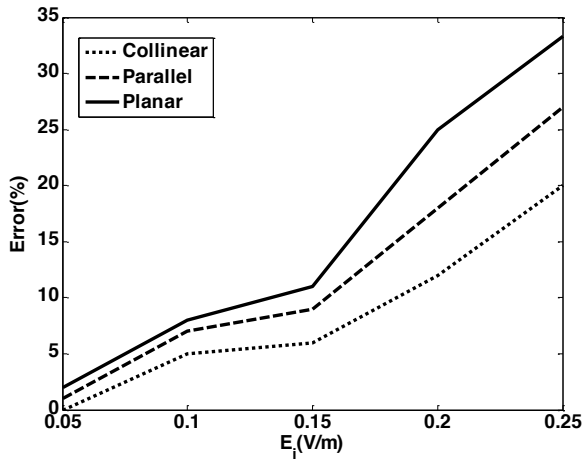
### V. SENSITIVITY ANALYSIS

In despite of efficiency of the approximate method, it leads to violated results when the magnitude of excitation,  $E_i$ , and nonlinearity of the load increased. For instance, Fig. 13 shows magnitude of the induced voltage across the p-n junction diode in the infinite planar array of nonlinear antennas under double-tone excitation with  $E_i = 0.25\text{V/m}$ . Comparison of Figs. 10(c) and 13 shows the mentioned fact. The maximum value of the relative error of the approximate method in Fig. 13 is 33.33% at 160 MHz which is not ignorable in practical applications.

To illustrate the effect of magnitude of incident wave on the maximum relative error, sensitivity analysis are also carried out on the magnitude of incident wave as shown in Fig. 14.



**FIGURE 13.** Magnitude of induced voltage of infinite planar array of nonlinear antennas under double-tone excitation by the NC and HB techniques under assumption of  $E_i = 0.25 \text{ V/m}$ .

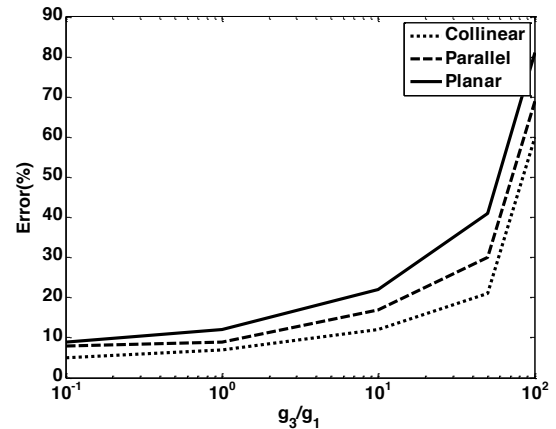


**FIGURE 14.** Maximum value of the relative error of the approximate method versus  $E_i$  for parallel, collinear and planar infinite arrays.

In Fig. 14, the maximum values of the relative errors for different infinite arrays of nonlinear antennas under double-tone excitation versus the magnitude of the incident wave ( $E_i$ ) are shown which can be useful in practical applications. In this figure, the nonlinear load is the p-n junction diode as characterized in (24). From this figure, when the magnitude of exciting wave is increased, the relative error percentage is also increased since the short circuit current in the microwave equivalent circuit is proportional to  $E_i$ . In addition, it can be found that the highest and lowest values of relative error percentage are respectively related to the planar and collinear arrays which are physically expectable if the effects of neighboring antennas or mutual coupling in the array are modeled based on the expansion wave concept [27]. Based on this concept, the effect of each neighboring antenna is modeled as an incoming wave. The magnitude of incoming waves for different arrangements is different so that the lowest and highest-valued couplings are respectively related to collinear and planar arrays [28].

A sensitivity analysis on the nonlinearity of the load can finally be carried out. To this aim, the nonlinear load as (26) is first assumed. The maximum relative error percentages versus  $g_3/g_1$  under assumption of  $E_i = 1 \text{ V/m}$  for the three infinite arrays of nonlinear antennas are then extracted as shown in Fig. 15. From this figure, it can be found that when  $g_3/g_1 \leq 1$ , the approximate method with maximum value of relative error percentage 10% can be useful.

$$i = g_1 v + g_3 v^3 \quad (26)$$



**FIGURE 15.** Maximum value of the relative error of the approximate method versus normalized nonlinearity ( $g_3/g_1$ ) for infinite parallel, collinear and planar arrays.

## VI. CONCLUSION

In this paper, validity of an efficient approximate modeling approach based on the nonlinear current technique for computation of scattering response from different infinite arrays of nonlinear antennas was investigated. Due to nonlinear nature of the problem under consideration, it is strongly dependent on nonlinear degree of the load and incident wave magnitude. Therefore, the valid ranges for the two mentioned parameters were extracted, which can be advantageous for practical applications such as adaptive phased arrays and frequency selective surfaces.

As another study, an efficient model for Norton's equivalent circuit for different infinite arrays based on qualitative concepts in electromagnetics the same as finite array [5] is underway.

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